Parametric array in air*

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An experimental investigation of the parametric array in air was conducted using a circular piston transducer which produced spherically spreading, collinear, primary beams at frequencies of 18.6 and 23.6 kHz. Since source levels were not strong (about 110 dB *re* 0.0002 μbar at 1 ft), the 5-kHz difference frequency signal generated by the parametric array was relatively weak. Because of space limitations, all measurements were made in the nearfield of the array. Spurious difference frequency signals resulting from intermodulation distortion in the receiving system were suppressed by judicious choice of electronic components and by the addition of an acoustical filter in front of the microphone. The classic properties of the parametric array were observed. The 5-kHz beam was narrow, and no minor lobes were evident. The propagation curve first increased with increasing range, reached a broad maximum, and then gradually decreased. Theoretical predictions were based on a perturbation solution of Burgers' equation and on the integral solution of the inhomogeneous wave equation. Comparison of measured results with these predictions conclusively demonstrated the existence of the parametric array in air. Beam patterns and propagation data obtained for the second-harmonic and sum-frequency signals also confirmed theoretical predictions.

Subject Classification: 25.35.

LIST OF SYMBOLS

a	effective radius of transducer face
B	$=(\gamma+1)\,r_0^2/4\gamma p_0 R$
c_0	small-signal sound speed
$D_n(\cdot)$	directivity function of nth component
f	frequency
k	wavenumber $(2\pi f/c_0)$
p_0	ambient atmospheric pressure
$p_n(R)$	peak pressure of nth component at range R
P_n	peak pressure of nth component at range r_0
R	range variable
r_0	selected reference range, roughly the point at which spherical spreading begins
R'	$=R-r_0$
R_0	Rayleigh distance ($ka^2/2$ for a circular piston)
α_n	small-signal absorption coefficient for the nth component
α_T	$=\alpha_1+\alpha_2-\alpha_d$
γ	ratio of specific heats
θ	angular deviation from axis of symmetry
θ_{HP}	beamwidth as determined by the half-power points
λ	wavelength

INTRODUCTION

In the period since Westervelt[1] first described the parametric acoustic array, the underwater parametric array has been the subject of numerous theoretical and experimental studies,[2] beginning with the measurements by Bellin and Beyer.[3] By comparison the parametric array in air has largely been ignored. At the time that our investigation was initiated (early 1972), data from only two experiments regarding the parametric array in air were available. In their study of the parametric array in water, Bellin and Beyer[3] also reported three data points which related the directivity of a difference frequency signal in air to the difference frequency itself. However, no data were obtained with respect to the propagation characteristics of the difference frequency signal. Brinkmann,[4] who was primarily interested in checking plane-wave theory, measured some propagation curves for a difference frequency signal in air but obtained no directivity patterns. It is of interest to note that Konrad and Mellen[5] have developed some design curves for the parametric array in air. These design curves were adapted from similar curves for the parametric array in water[6] by substituting the appropriate parameters of air for those of water. Konrad and Mellen show good agreement between a prediction of extrapolated difference frequency source level obtained from their design curves and the corresponding data point from the Brinkmann experiment. However, this one data point appears to be the only valid one available with which Konrad and Mellen could check their design curves.

Thus it is seen that only a very limited amount of experimental data for the parametric array in air was available. Moreover, there had been reports of an unsuccessful parametric-array-in-air experiment.[7] The controversy over the very existence of the parametric array in air was, in fact, the primary motivation for our undertaking the experiment reported here.

We sought first to demonstrate that the parametric array does indeed work in air. If we were successful, we then wished to acquire quantitative data for both the directivity and propagation characteristics of the array. Because it was relatively simple to do so, we also obtained data for the sum frequency and second harmonic signals. Although the investigation was essentially experimental, some theoretical calculations were made. The theoretical approaches utilized are summarized below but are explained more fully in Ref. 8, wherein further details of the entire investigation may be found.

Motivated in much the same way, Shealy and Eller independently performed a similar experiment at about the same time. The results of both investigations were reported at the 85th Meeting of the Acoustical Society of America, April 1973.[9,10] Subsequently, Widener and Muir reported the results of another airborne experiment.[11]

The important conditions of our experiment were as follows:

(1) The zone of interaction was formed by spherically spreading primary beams.

(2) Each primary beamwidth was about three times the computed Rutherford scattering beamwidth.[1]

(3) Attenuation of the primary signals was due to small-signal absorption, not finite-amplitude effects.

(4) Because of space limitations, the data were taken in the nearfield of the parametric array.

I. THEORY

Two approaches used to obtain theoretical predictions are described. First is the integral solution of the inhomogeneous wave equation.[1] This approach is rigorous in that effects of diffraction are properly accounted for, but the integration is difficult to carry out. Westervelt's classical results[1] are obtained if the primary beams are sufficiently narrow and if the observation points are in the farfield of the parametric array. Unfortunately, neither of these requirements is met in our experiment. However, the integral solution may be evaluated numerically. We used the Muir–Willette method,[12] which was developed for spherically spreading primary beams generated in water by a circular piston. The adaptation of the computer program to airborne parametric arrays is straightforward. Unfortunately, the numerical routine takes a rather large amount of computer time, especially for the beam patterns. For this reason beam patterns were computed only for the range $0 \le \theta \le 24°$. As noted below, it was not necessary to use the numerical routine for the sum-frequency or second-harmonic components.

In the second approach, the model used is Burgers' equation for directional, spherically spreading beams.[8] A second-order perturbation solution of the Burgers equation for this case yields the following expressions for the amplitudes of the various second-order components[8]:

$$p_{2f_1}(R,\theta) = Bk_1 P_1^2 D_1^2 e^{-4\alpha_1 R'} \int_{r_0}^{R} \frac{e^{2\alpha_1 x'}}{x}\, dx\ , \qquad (1)$$

$$p_{2f_2}(R,\theta) = Bk_2 P_2^2 D_2^2 e^{-4\alpha_2 R'} \int_{r_0}^{R} \frac{e^{2\alpha_2 x'}}{x}\, dx\ , \qquad (2)$$

$$p_s(R,\theta) = Bk_s P_1 P_2 D_1 D_2 e^{-\alpha_s R'} \int_{r_0}^{R} \frac{e^{\alpha_T x'}}{x}\, dx\ , \qquad (3)$$

$$p_d(R,\theta) = Bk_d P_1 P_2 D_1 D_2 e^{-\alpha_d R'} \int_{r_0}^{R} \frac{e^{-\alpha_T x'}}{x}\, dx\ , \qquad (4)$$

where $x' = x - r_0$. The subscripts $2f_1$, $2f_2$, s, and d denote the lower second harmonic, the higher second harmonic, the sum frequency, and the difference frequency, respectively. If desired, the integrals in these equations may be expressed in terms of exponential integral functions. Note that, strictly speaking, Burgers' equation is for a thermoviscous medium, whereas air is a relaxing medium. Nevertheless, Burgers' equation is still a valid model for air provided the absorption coefficient α is proportional to f^2 over the frequency range of interest. In our experiment, the f^2 dependence held reasonably well.

The dependence of axial amplitude on range appears to be a relatively complicated function in Eqs. 1–4. A useful approximation can be obtained, however, by taking the limit as the absorption coefficients vanish. Then the range dependence reduces to $R^{-1}\ln(R/r_0)$ for each of the components. Thus one expects the slope of the propagation curve to approach 6 dB per doubling of range in the farfield of the parametric array, but to be less than that in the nearfield. It should be noted here that the farfield of an absorption-limited parametric array is generally considered to begin at approximately $1/2\alpha_0$,[6] where α_0 is the absorption coefficient corresponding to the center frequency of the primary waves.

With regard to the directivity characteristics implied by Eqs. 1–4, we see that the beam pattern of each combination tone is given by either the square or the product of the primary patterns. Although such a prediction works well for the second-harmonic and sum-frequency signals, the product of the primary beam patterns cannot be used to describe the difference frequency beam pattern unless the half-power beamwidth associated with the product pattern is much greater than the Rutherford scattering beamwidth.[2] The product pattern in our experiment did not satisfy this condition.

II. APPARATUS

A schematic diagram of the equipment employed is shown in Fig. 1. The primary frequencies used were 18.6 and 23.6 kHz, so that the difference frequency was 5 kHz. The experiment was conducted in a small test room (8 ft×8 ft×9 ft) that had been rendered anechoic above the middle audio range by covering walls, ceiling, and floor with four or more inches of fiberglass insulation material. The propagation measurements were taken along a horizontal diagonal of the room approximately 4 ft above the floor. Because of the physical size of the transmitter with its turntable and of the mobile microphone assembly, the maximum useful measurement range along this diagonal was about 7 ft. Pure-tone propagation measurements made in the room showed that anechoic conditions prevailed over the 7-ft measurement path down to about 3.5 kHz. The ambient acoustical noise in the room was found to be at least as low as 22 dB *re* 0.0002 μbar in any 1/3-octave band above 500 Hz. Equipment inside the room was kept to a minimum. The sound source with its associated turntable, a monitoring microphone, the receiving microphone, and a small two-channel microphone power supply were the only pieces of equipment inside the

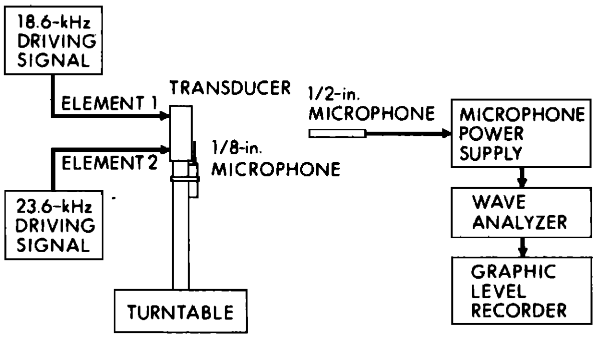

FIG. 1. Schematic of measurement apparatus.

room. A trolley system suspended from the ceiling allowed easy movement of the receiving microphone to any desired range point.

The sound source was a two-element squirter transducer originally designed for underwater transmission.[13] The active face of the transducer was circular and had a diameter of about 2.5 in. The primary frequencies were chosen to coincide with resonances of the elements. In Fig. 2, the measured beam patterns for the squirter transducer operating at the primary frequencies are compared to the beam patterns of an ideal piston with a 2.5-in-diameter face operating at the same frequencies. The ideal piston pattern is too narrow in one case and too broad in the other. In order to model the experimental conditions as closely as possible, the ideal patterns were required to match the measured patterns at the half-power points. Imposing this requirement gives an average effective transducer radius a of 1.15 in. This value is the one used in subsequent theoretical calculations. Since the mean Rayleigh distance ($k_0a^2/2$, where k_0 is the mean primary wavenumber) was $R_0 = 0.536$ ft, less than 10% of the available measurement path was taken up by the nearfield of the primary waves. The maximum rms sound pressure level (SPL) produced at 1 ft by each element was about 110 dB *re* 0.0002 μbar. The electrical inputs to the two elements were monitored to ensure purity of signal. In addition, a Brüel & Kjær (B&K) type 4138 1/8-in. microphone was mounted at the face of the sound source to monitor the acoustic output.

The requirements for the receiving microphone were somewhat contradictory. Because the 5-kHz difference frequency tone was weak, high sensitivity was desired. At the same time, extended high-frequency response was necessary in order to measure the primary, second-harmonic, and sum-frequency signals. The B&K type 4133 1/2-in. microphone selected as the receiver represented a compromise biased toward the difference frequency measurement. A General Radio (GR) type 1900-A wave analyzer, operated in the 50-Hz bandpass mode, was used to measure the levels of the various components of the received signal, and a GR type 1521-B graphic level recorder, synchronized with the turntable, was employed to record the beam patterns.

Considerable care was taken to ensure that extraneous difference frequency signals from various sources did not interfere with the measurement of the difference frequency signal generated by the parametric array. Fortunately, these spurious signals have readily distinguishable properties. As noted in Sec. I, the parametric-array signal varies with range approximately as $R^{-1}\ln(R/r_0)$. Such a dependence implies a propagation curve that first rises, reaches a broad maximum, and then gradually decreases, approaching a slope of 6 dB per doubling of range as the farfield of the parametric array is reached. Difference frequency signals generated by intermodulation distortion in the receiving system vary as the product of the local primary amplitudes at the receiver, and thus define an apparent propagation curve whose slope is 12 dB per doubling of range. A 12-dB slope is also characteristic of pseudosound, which is a difference frequency signal produced at the face of the receiver by interaction radiation pressure. Spurious difference frequency radiation originating in the transmitting system can be identified by its broad-beam directivity, which is characteristic of a piston excited at a low frequency.

Prior to experimentation, intermodulation distortion in the receiving system electronics was investigated. Electrical signals equivalent to those transduced by the microphone at the primary frequencies were impressed directly upon the microphone preamplifier. The difference frequency distortion was found to be 87 dB below the level of the primary signals. However, when a parametric-array measurement was attempted, spurious difference frequency signals, identified by the steep slope of the apparent propagation curve, were still observed. The transmitting system was eliminated as a possible source of the spurious signals because the difference frequency beam pattern was quite narrow. Pseudosound was eliminated on the basis of the strength of the spurious signals. Thus we concluded that the distortion was most likely occurring in the microphone cartridge. As a result, a dome-shaped acoustic filter to enclose the microphone head was constructed from a 2.5-mil-thick clear plastic material. This acoustic filter had a low-pass characteristic; it reduced the SPL of the difference frequency signal at the receiving mi-

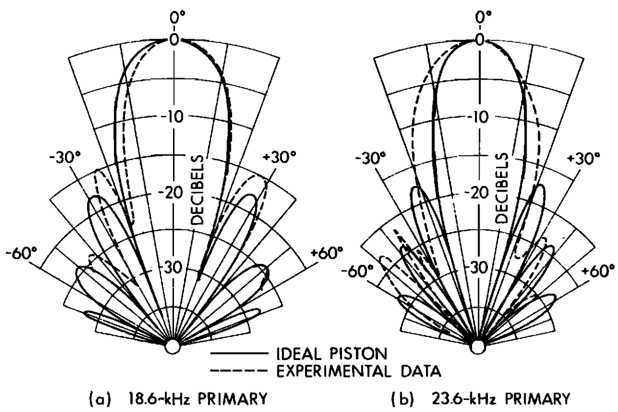

FIG. 2. Comparison of measured and ideal piston beam patterns for the two primary signals. Measured θ_{HP} of 18.6-kHz signal: 16.2°. Measured θ_{HP} of 23.6-kHz signal: 16.6°.

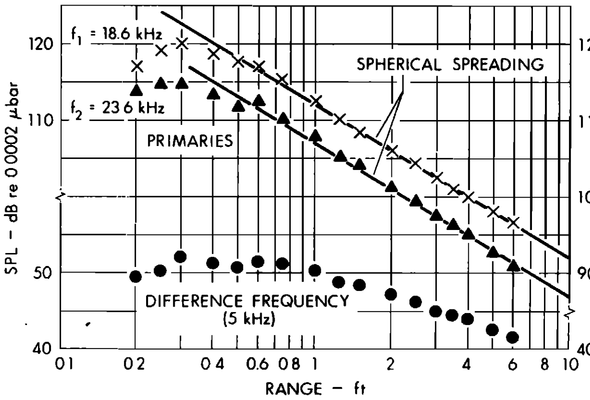

FIG. 3. Measured propagation curves: filtered primary and difference frequency components.

crophone by only 3.5 dB, but attenuated the primary signals by approximately 20 dB.

The filter was found to be an effective means of reducing the level of the spurious signals so that the parametric-array-generated difference frequency signal was not masked. A filter is not necessary for measuring the high-frequency combination tones because these components are much stronger than the difference frequency signal (see Eqs. 1–4). Spurious signals compete much more effectively with the difference frequency signal than with any of the other components.

With regard to the question of experimental error, accuracy of the amplitude measurements was good at the lower frequencies but deteriorated somewhat as frequency increased. Errors in the difference frequency data are estimated to be no more than ± 1 dB. The accuracy at the primary frequencies is estimated to be ± 2 dB. Errors in the high-frequency data were higher because of the rapid variation with frequency of the receiver response (mainly the microphone and the acoustic filter) in this frequency range. On the other hand, the directivity patterns do not depend upon absolute amplitudes and are felt to be reasonably accurate at all of the measured frequencies.

III. RESULTS AND DISCUSSION

The experimental propagation data obtained with the primaries acoustically filtered at the microphone are shown in Fig. 3. The two primary signals exhibit the expected spherical spreading reduction of 6 dB per doubling of range, and the difference frequency data display the classic parametric-array propagation characteristics described in the previous sections. Because of the physical size of our test room, our measurements were limited to the nearfield of the array. Thus, the slope of the difference frequency propagation curve never reaches the expected farfield value of 6 dB per doubling of distance. Approximately 30 ft would have been required to establish the farfield of the array.

In Fig. 4, the theoretical propagation curves for the difference frequency component are compared with the experimental data. The following values of constants were used to make the theoretical calculations: $c_0 = 1130$ ft/sec, $p_0 = 29.30$ in. Hg, $\gamma = 1.4$, $\alpha_T = 0.0345$ ft^{-1}, and $\alpha_d = 0.00115$ ft^{-1} (absorption coefficients were obtained from Ref. 14; the relative humidity was 50%). In the numerical integration computation, both in this instance and in the calculation of beam patterns, an ideal piston pattern (with the effective value $a = 1.15$ in. used for the piston radius) was used for each primary directivity function. The value 0.13 ft ($0.75R_0/\pi$) was used as the lower limit in the integration over the range variable[12] (taking the lower limit to be zero produced essentially the same results). In order to make a computation based on the perturbation solution, Eq. 4, it is necessary to specify the effective source radius r_0. Past experience[15,16] in using piston sources to make finite-amplitude measurements in water has shown that, except close to r_0, good agreement between theory and experiment is achieved if r_0 is taken to be in the range $R_0/3 < r_0 < 3R_0/4$. In Fig. 4, curves for $r_0 = 0.27$ ft ($R_0/2$) and $r_0 = 0.40$ ft ($3R_0/4$) are shown.

Although the perturbation solution with $r_0 = 3R_0/4$ seems to provide a superior fit to the measured data than the numerically evaluated integral solution, the superiority is not deemed significant. If each primary amplitude was actually 1.5 dB higher than the value recorded (an amount easily within the experimental error), all the theoretical curves should be raised 3 dB, and the integral solution then provides the best fit. Incidentally, the fact that the perturbation solution curves are higher than the integral solution curve is to be expected. In the Burgers equation model no account is taken of the reduction in difference frequency output caused by diffraction.

Unfortunately, the restriction of the difference frequency measurements to the deep nearfield of the parametric array makes it very difficult to accurately extrapolate the measurements to the farfield. We therefore make no attempt to estimate the source level of the array. By adjusting the values of P_1 and P_2 so that the integral solution curve gives a good fit to the measured data, we could in principle obtain a theoretical extrapolation to the farfield and thus obtain a source level estimate. Such a procedure would at best be suspect, however, and in any case is beyound the scope of the present study. The main purpose of the experiment was to determine the existence of the parametric array in air, not to test various design models of the array.

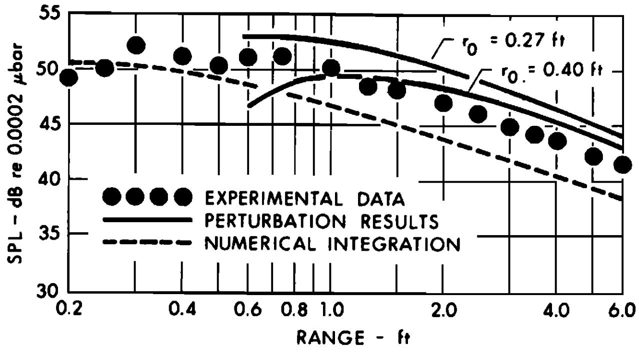

FIG. 4. Comparison of measured and computed difference frequency propagation curves. SPL_1(1 ft) = 112 dB *re* 0.0002 μbar and SPL_2(1 ft) = 107 dB *re* 0.0002 μbar.

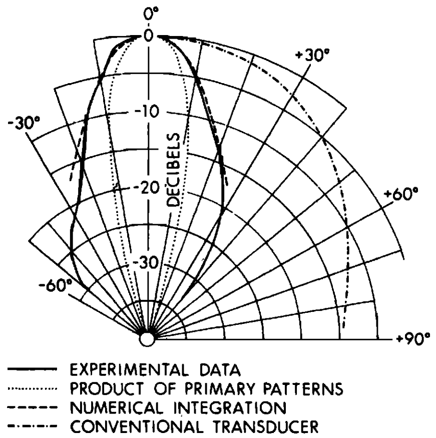

FIG. 5. Comparison of measured and computed difference frequency beam patterns at a range $R=2$ ft. Experimental data: $\theta_{HP}=16.0°$; product of primary patterns: $\theta_{HP}=11.4°$; numerical integration: $\theta_{HP}=16.0°$; conventional transducer: $\theta_{HP}=60.0°$.

Propagation data for the sum-frequency and second-harmonic tones are not given here but may be found in Ref. 8. In general the data define curves having the same shapes as the corresponding curves computed from Eqs. 1–3. Although agreement as to absolute amplitude leaves something to be desired (agreement is good, fair, and poor for the $2f_1$, f_s, and $2f_2$ tones, respectively), allowance must be made for the fact that measurement errors grew rather rapidly in this frequency region; see the discussion at the close of Sec. II. Overall, we conclude that the data confirm the perturbation solution of Burgers' equation for these tones.

Theoretical and experimental beam patterns for the difference frequency signal at a range of 2 ft are shown in Fig. 5. To emphasize the striking contrast between the directional properties of a conventional transducer and the parametric array, we have also plotted a beam

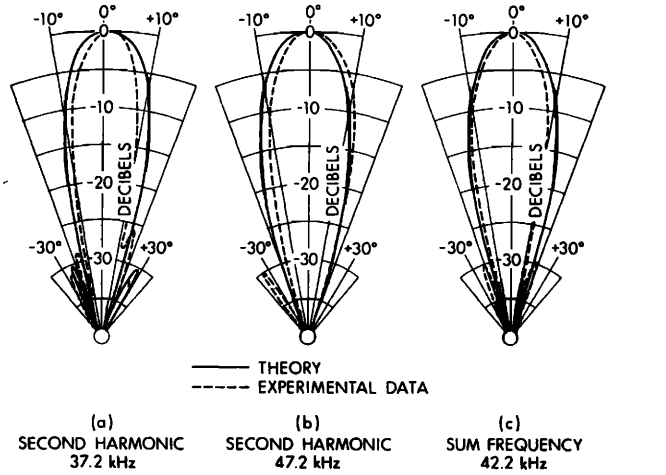

FIG. 6. Comparison of measured and computed beam patterns for the second-harmonic and sum-frequency components at a range $R=1$ ft. Measured beamwidths: 37.2-kHz second harmonic, $\theta_{HP}=9.0°$; 47.2-kHz second harmonic, $\theta_{HP}=11.5°$; 42.2-kHz sum frequency, $\theta_{HP}=9.5°$. Computed beamwidths: 37.2-kHz second harmonic, $\theta_{HP}=12.4°$; 47.2-kHz second harmonic, $\theta_{HP}=11.2°$; 42.2-kHz sum frequency, $\theta_{HP}=10.0°$.

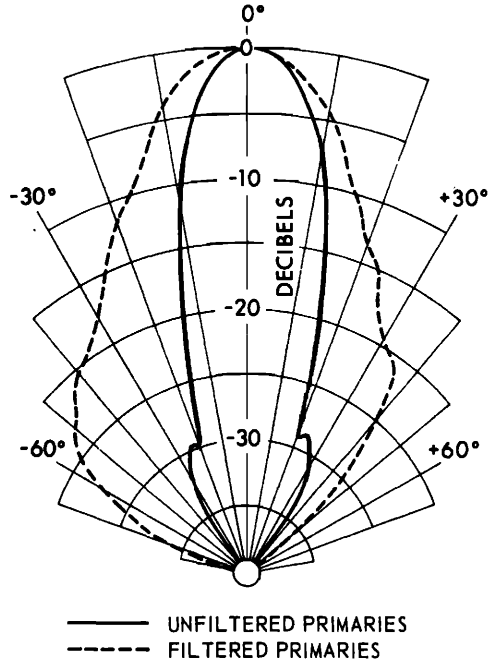

FIG. 7. Comparison of measured difference frequency patterns for filtered and unfiltered primary beams at a range $R=1$ ft.

pattern that would be observed if the transducer itself were driven at the difference frequency. The actual measured primary directivities (dashed patterns in Fig. 2) were used to compute the product pattern. The measured beam pattern closely confirms the theoretical pattern obtained by means of the Muir–Willette numerical integration technique. Close confirmation was also found for patterns taken at $R=1$ ft and $R=4$ ft; as expected, the beamwidth decreases slightly with range. The good agreement with respect to beam patterns is evidence that little significance should be attached to the 3-dB difference between the measured and numerically computed propagation curves in Fig. 4. Incidentally, it should be noted that a mistake was made in Ref. 8 in computing the beam pattern by the numerical integration method. As a result, the theoretical pattern attributed to this method in Ref. 8 (Fig. 22) and in Ref. 9 is too broad. [17]

Theoretical and experimental beam patterns for the second-harmonic and sum-frequency tones are shown in Fig. 6. The theoretical patterns in this case are product patterns, as given in Eqs. 1–3. Agreement is satisfactory.

We now turn to a brief discussion of the experimental aspects of spurious difference frequency signals. Difference frequency beam patterns with and without acoustical filtering at the microphone are shown in Fig. 7. Without filtering, the pattern is quite sharp and, in fact, matches the theoretical prediction based on Eq. 4, i.e., a product pattern. However, the product pattern is also produced when the source of difference frequency signals is either pseudosound or intermodulation distortion in the receiver. Thus, narrow beam patterns are not proof by themselves of the existence of the parametric array. They only eliminate the transmitter

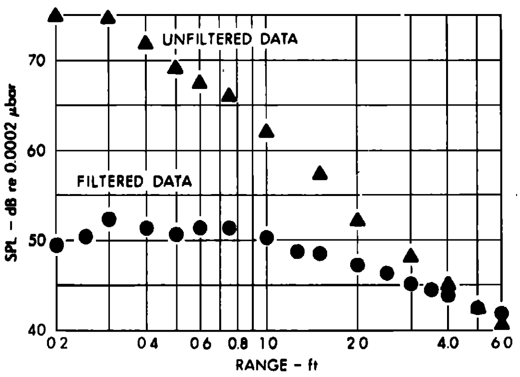

FIG. 8. Comparison of filtered and unfiltered data for difference frequency propagation curves.

as a possible source of spurious difference frequency signals. To further identify the source of narrow-beam signals, one must measure the propagation curve. Figure 8 shows propagation data with and without acoustical filtering. The unfiltered data clearly show contamination by spurious signals, in this case determined to be caused by receiver distortion. Only after filtering is the true parametric-array curve revealed. The narrow pattern in Fig. 7 is thus identified as spurious. Incidentally, the slope of the unfiltered data curve is approximately 8 dB per doubling of distance, which is somewhat less than the 12-dB slope that would be expected from spurious signals alone. In this case the observed curve represents a mixture of spurious and true parametric-array signals. At short range the spurious signals predominate, whereas at longer range parametric-array radiation takes over.

Yet another difficulty deserves comment. Experimenters since the time of Thuras, Jenkins, and O'Neil[18] have been troubled by the exaggerated influence seemingly small errors have when comparisons are made between theory and measurement in nonlinear acoustics. We have already noted the 3-dB discrepancy between the measured and theoretical difference frequency propagation curves. Although a 3-dB discrepancy may seem large, to obtain the two curves, one must make three absolute amplitude measurements, one for each primary and one for the difference frequency component. Thus, a seemingly trivial error of 1 dB in each measurement could account for the entire discrepancy. Small errors in the values used for common constants can also have an insidious effect on comparisons between theory and experiment. The amplitude factor in the integral solution contains the sound speed c_0 raised to the fourth power (see Eq. 5 in Ref. 12). Thus an error in the value used for c_0 of 30 ft/sec (i.e., less than 3%) causes an error of 1 dB in the computed difference frequency SPL. It was found that use of the effective transducer radius $a = 1.15$ in. in place of the actual face radius 1.26 in. resulted in a 1-dB change in the computed SPL. Fortunately, the beam-pattern calculations are not sensitive to amplitude factors and are therefore less subject to magnified errors. In general, however, considerable care must be taken to be sure that a discrepancy (or agreement!) between a theoretical prediction and an experimental measurement is indeed significant.

IV. CONCLUSIONS

The experimental results prove that the parametric array does indeed work in air, despite the limitation of the measurements to the nearfield of the array. The filtered data for the difference frequency beam pattern and propagation curve have the characteristics that are expected qualitatively for an absorption-limited array formed by spherically spreading primary beams. Quantitatively, these data give a reasonable confirmation of the predictions based on the integral solution (numerically evaluated by the Muir–Willette method) of the inhomogeneous wave equation. In the case of the sum-frequency and second-harmonic components, however, the predictions obtained from the perturbation solution of Burgers' equation are sufficient to explain the measured beam patterns and propagation curves.

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