Proceedings of Meetings on Acoustics

Volume 19, 2013 http://acousticalsociety.org/

ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013

Signal Processing in Acoustics
Session 1pSPa: Nearfield Acoustical Holography (NAH) Measurements and Applications

1pSPa6. Improved hydrophone calibration by combining acoustic holography with the radiation force balance measurements

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Ultrasound sources are frequently characterized by the radiation force (RF) balance method that is based on the relation between the total acoustic power and RF on absorbing or reflecting targets. This relation is usually taken from the plane-wave approximation or with a geometrical correction for focused sources. However, real sources emit inhomogeneous acoustic beams. Acoustic holography is a method of recording the true field by measuring both pressure magnitude and phase over a 2D surface (a hologram). The hologram makes it possible to accurately calculate the radiation stress tensor on the surface of the absorbing target. Such measurements allow the relation of hydrophone sensitivity with measured RF based on the known exact expression for RF as a function of the 2D pattern of acoustic pressure and particle velocity. This suggests an improved approach for single-frequency hydrophone calibration that benefits from the inherent accuracy of mass balances as well as the fact that error in the pressure calibration scales as half of the RF measurement error. In the current study this approach was used to calibrate a hydrophone by characterizing a 1-MHz focused piezoceramic source in water using acoustic holography and a RF balance.

Published by the Acoustical Society of America through the American Institute of Physics

Received 22 Jan 2013; published 2 Jun 2013

INTRODUCTION

Measurement of ultrasonic power is required by international standards for both diagnostic and therapeutic ultrasound systems [1]. The total acoustic power of an ultrasound source is typically found by measuring the radiation force acting on an absorbing target that fully intersects the acoustic beam. The relation between total power and measured radiation force is usually presumed from a plane-wave approximation [2] or estimated with a geometrical correction for focused sources [3]. However, in reality the acoustic beam can be far from a plane wave and the source can vibrate much differently than a uniform "piston." Therefore, it can be very important to account for the true structure of the acoustic pressure field in relating radiation force to acoustic power. An efficient method for considering the structure of an arbitrary acoustic beam is to characterize the radiated field using acoustic holography, which involves measurements of pressure magnitude and phase over a 2D surface (a hologram) [4—6]. The hologram may be used to relate the total power and the radiation force with much better accuracy than that of currently existing approaches.

RADIATION FORCE ON AN EXTENDED ABSORBING TARGET AND ITS RELATION TO THE TOTAL ACOUSTIC POWER

Consider an ultrasound beam with a sinusoidal time dependence of acoustic pressure $\tilde{p}(\mathbf{r},t)$ and particle velocity $\tilde{\mathbf{v}}(\mathbf{r},t)$:

$$\tilde{p} = \frac{p}{2}e^{-i\omega t} + \frac{p^*}{2}e^{i\omega t}, \qquad \tilde{\mathbf{v}} = \frac{\mathbf{v}}{2}e^{-i\omega t} + \frac{\mathbf{v}^*}{2}e^{i\omega t} \tag{1}$$

Here $p(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$ are complex amplitudes, the asterisks denote complex conjugation, and $\omega/(2\pi)$ is the source excitation frequency. The radiation force $\mathbf{F}$ on an object surrounded by a closed surface Σ can be expressed as follows [7]:

$$\mathbf{F} = \oiint_{\Sigma} \frac{d\mathbf{F}}{d\Sigma} d\Sigma \tag{2}$$

$$\frac{d\mathbf{F}}{d\Sigma} = \left\langle \left(\frac{\rho \tilde{\mathbf{v}}^2}{2} - \frac{\tilde{p}^2}{2\rho c^2} \right)\mathbf{n} - \rho \tilde{\mathbf{v}}\,(\tilde{\mathbf{v}} \cdot \mathbf{n}) \right\rangle \tag{3}$$

Here, $\mathbf{n}$ is the external normal unit vector to surface element $d\Sigma$, the brackets $\langle ... \rangle$ denote averaging over a wave period, and and c are density and sound speed, respectively. From Eqs. (1) and (3) we obtain:

$$\frac{d\mathbf{F}}{d\Sigma} = \left(\frac{\rho |\mathbf{v}|^2}{4} - \frac{|p|^2}{4\rho c^2} \right)\mathbf{n} - \frac{\rho}{2}\mathrm{Re}\left[\mathbf{v}^*(\mathbf{v}\cdot\mathbf{n})\right] \tag{4}$$

Consider now the acoustic power that goes into the volume limited by the surface Σ:

$$W = \oiint_{\Sigma} \mathbf{I}\cdot\mathbf{n}\, d\Sigma \tag{5}$$

Here $\mathbf{I}$ is acoustic intensity vector – *i.e.*, the Umov-Poynting vector averaged over the wave period:

$$\mathbf{I} = \langle \tilde{p}\tilde{\mathbf{v}} \rangle = \frac{1}{2}\mathrm{Re}(p\mathbf{v}^*) \tag{6}$$

Note that by considering the linearized equation of motion, $\rho\, \partial\tilde{\mathbf{v}}/\partial t = -\nabla\tilde{p}$, the complex amplitude of the harmonic wave is expressed through acoustic pressure gradient:

$$\mathbf{v} = \nabla p/(i\rho\omega) \tag{7}$$

To represent the conditions typically present in radiation force balances, an ideal case can be considered in which the surface Σ surrounds an absorber that is sufficiently wide in the lateral direction to "catch" the entire acoustic beam, and sufficiently thick to make the acoustic field negligible on the back side. For such a case, the integration over the closed surface Σ is reduced to the integration over the face of the absorber. Let us consider this face as an xy-plane surface located at $z = z_0$. Then the external normal is $\mathbf{n} = (0,0,-1)$ and a radiation force balance measures the z-component of the radiation force. We can therefore write:

$$F_z = \frac{\rho}{4} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dxdy \left\{ \frac{|p|^2}{\rho^2 c^2} + |v_z|^2 - |v_x|^2 - |v_y|^2 \right\} \tag{8}$$

$$W = \frac{1}{2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dxdy \, \mathrm{Re}\left(p v_z^*\right) \tag{9}$$

Note that the integration here can be performed not only on the absorber face $z = z_0$, but along any xy-plane in front of the source, because of the absence of momentum loss when the wave is propagating in an inviscid medium [7]. From Eqs. (8) and (9), note also that the simplified relation between the force and total power, $F_z = W/c$, is valid only for a normally incident plane wave, for which $v_z = p/(\rho c)$ and $v_x = v_y = 0$.

USE OF ACOUSTIC HOLOGRAPHY AND ANGULAR SPECTRUM

The full characterization of an acoustic source can be made with acoustic holography – *i.e.*, recording of the lateral distribution of acoustic pressure magnitude and phase at some surface in front of the transducer. With such measurements, a 2D distribution of the complex amplitude $p(x, y, z_H)$ is known at rectangular grid points in the measurement plane $z = z_H$. There are two possible ways of using the measured distribution to calculate the radiation force and total power. The most obvious is to use $p(x, y, z_H)$ to calculate acoustic field parameters p and $\mathbf{v}$ at the absorber face $z = z_0$, as could be accomplished using a Rayleigh integral approach [4]. Although we have used such an approach, here we present an alternative that uses an angular spectrum representation of the pressure measurements. As such, the acoustic field can be written in terms of the angular spectrum $S(k_x, k_y)$ as follows:

$$p(x, y, z) = \frac{1}{4\pi^2} \iint_{k_x^2 + k_y^2 \le k^2} dk_x dk_y S(k_x, k_y) e^{ik_x x + ik_y y + i\sqrt{k^2 - k_x^2 - k_y^2}\,(z - z_H)} \quad , \tag{10}$$

where k_x and k_y are the *x*- and *y*-components of the wave vector, and $k = \omega/c$ is the wavenumber.

Here the angular spectrum is calculated based on the complex pressure amplitude distribution in the measurement plane (the hologram plane $z = z_H$):

$$S(k_x, k_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dxdy \, p(x, y, z_H) e^{-ik_x x - ik_y y} \quad . \tag{11}$$

Considering Eq. (7), we can derive all the particle velocity components:

$$\begin{aligned}
v_x(x, y, z) &= \frac{1}{4\pi^2} \iint_{k_x^2 + k_y^2 \le k^2} dk_x dk_y \frac{k_x/k}{\rho c} S(k_x, k_y) e^{ik_x x + ik_y y + i\sqrt{k^2 - k_x^2 - k_y^2}\,(z - z_H)} \\
v_y(x, y, z) &= \frac{1}{4\pi^2} \iint_{k_x^2 + k_y^2 \le k^2} dk_x dk_y \frac{k_y/k}{\rho c} S(k_x, k_y) e^{ik_x x + ik_y y + i\sqrt{k^2 - k_x^2 - k_y^2}\,(z - z_H)} \\
v_z(x, y, z) &= \frac{1}{4\pi^2} \iint_{k_x^2 + k_y^2 \le k^2} dk_x dk_y \frac{\sqrt{k^2 - k_x^2 - k_y^2}/k}{\rho c} S(k_x, k_y) e^{ik_x x + ik_y y + i\sqrt{k^2 - k_x^2 - k_y^2}\,(z - z_H)}
\end{aligned} \tag{12}$$

If we now substitute Eqs. (12) into Eqs. (8) and (9), change the order of integration, and account for the spectral representation of the Dirac delta function, we come to the following expressions for radiation force and total acoustic power:

$$F_z = \frac{1}{8\pi^2 \rho c^2} \iint_{k_x^2+k_y^2 \le k^2} \left(1 - \frac{k_x^2 + k_y^2}{k^2}\right) \left| S(k_x, k_y) \right|^2 \, dk_x dk_y \tag{13}$$

$$W = \frac{1}{8\pi^2 \rho c} \iint_{k_x^2+k_y^2 \le k^2} \sqrt{1 - \frac{k_x^2 + k_y^2}{k^2}} \left| S(k_x, k_y) \right|^2 \, dk_x dk_y \tag{14}$$

These equations show that although radiation force and total power are nonlinear acoustic quantities, each plane wave of the angular spectrum contributes to the net force and power independently, as if F_z and W were linear quantities. If the beam is a quasi-plane wave, then its angular spectrum $S(k_x, k_y)$ is localized near the origin of the spatial frequencies plane (k_x, k_y), and the expected plane-wave result $F_z = W/c$ follows from Eqs. (13) and (14). However, the relationship between F_z and W does not generally simplify to this trivial case.

In the expressions (13) and (14) it is presumed that the target is an ideal absorber. If this is not the case and a part of the incident acoustic energy is reflected from the absorber face, then Eqs. (13) and (14) can be easily modified based on known expressions for the reflection coefficient of an arbitrarily inclined plane wave from a plane interface. This may improve the accuracy of the radiation force balance measurements. However, if the wave reflection from the absorber is significant, the back-propagated wave would reflect from the transducer face and come back to the absorber. Such multiple reflections would complicate radiation force predictions and are inherently undesirable.

For an ideal absorber it is possible to express the radiation force through the measured 2D pressure distribution $p(x, y, z_H)$ directly, without calculating the angular spectrum. This can be done based on the following equality that holds when the "pushing" acoustic field consists of only forward-propagating angular spectrum components:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |\mathbf{v}|^2 \, dxdy = \frac{1}{\rho^2 c^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |p|^2 \, dxdy \tag{15}$$

The equality (15) follows directly from Eqs. (10) and (12). Using Eqs. (7), (8) and (15), radiation force can be expressed in a form that involves only pressure $p(x, y, z_H)$ and its lateral derivatives:

$$F_z = \frac{1}{2\rho c^2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dxdy \, \left\{ |p|^2 - \frac{|\partial p/\partial x|^2 + |\partial p/\partial y|^2}{k^2} \right\} \tag{16}$$

This representation of the radiation force through the measured pressure distribution $p(x, y, z_H)$ needs fewer computations than calculations based on Eqs. (11) and (13).

COMBINATION OF ACOUSTIC HOLOGRAPHY AND RADIATION FORCE BALANCE METHODS TO IMPROVE TOTAL POWER CALCULATIONS

Equations (11), (13), and (14) define an important relation between total acoustic power and radiation force through an experimentally recorded acoustic hologram $p(x, y, z_H)$. Let U be electrical voltage at the hydrophone induced by acoustic pressure p, and $M = U/p$ be the hydrophone sensitivity. Consider the angular spectrum for the hydrophone voltage signal:

$$S_U(k_x, k_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dxdy \, U(x, y) e^{-ik_x x - ik_y y} \tag{17}$$

Note that $S(k_x, k_y) = S_U(k_x, k_y)/M$. Equations (13) and (14) show that the ratio F_z/W does not depend on M, which means the total power can be calculated from the measured radiation force even if the pressure field

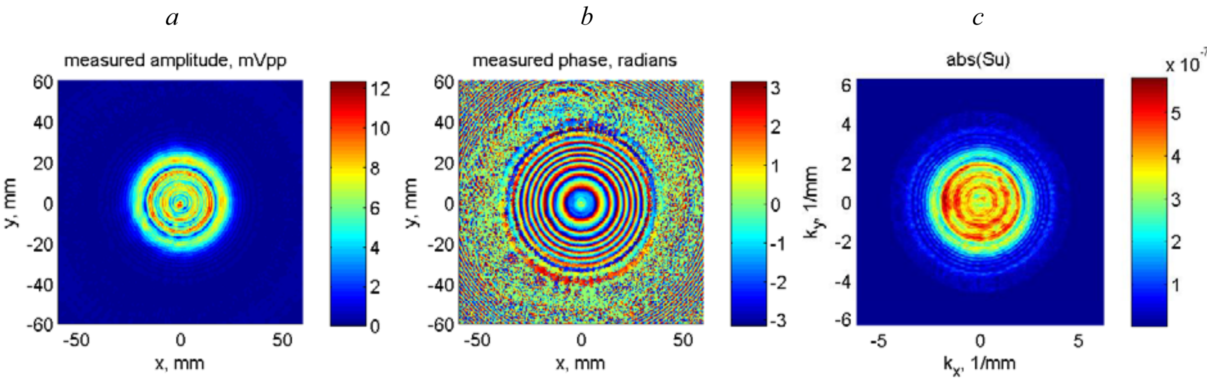

FIGURE 1. Pressure magnitude (*a*) and phase (*b*) measured at 50 mm from the source. The corresponding magnitude of the angular spectrum is shown at the right (*c*).

measurement is made with a hydrophone of unknown sensitivity. The corresponding relation can be written in the form

$$W = \gamma \, c F_z \tag{18}$$

where the factor γ accounts for the true structure of the beam and thus corrects the plane-wave approximation. In case of a plane wave $\gamma = 1$, but $\gamma > 1$ for the real beam:

$$\gamma = \frac{\iint\limits_{k_x^2+k_y^2 \le k^2} \sqrt{1 - \frac{k_x^2 + k_y^2}{k^2}} \, \left| S_U(k_x, k_y) \right|^2 dk_x dk_y}{\iint\limits_{k_x^2+k_y^2 \le k^2} \left(1 - \frac{k_x^2 + k_y^2}{k^2}\right) \left| S_U(k_x, k_y) \right|^2 dk_x dk_y} \tag{19}$$

To illustrate the importance of such a correction, we performed experimental characterization of a focused piezoceramic source. The source has a spherical shape with a 100 mm diameter and 100 mm radius of curvature. The operating frequency was 1.092 MHz. Figure 1 shows a hologram of this source (*i.e.*, pressure magnitude and phase) measured at 50 mm from the source center (*a* and *b*) and the corresponding magnitude of the angular spectrum (*c*). It is seen that the angular spectrum is distributed over wide region, which can be explained by the curved shape of the source and the presence of Lamb waves in the piezoceramic plane vibrations [4]. Application of Eq. (19) to the measured data yields the value of the correction factor $\gamma = 1.089$.

HYDROPHONE CALIBRATION BASED ON ACOUSTIC HOLOGRAPHY AND RADIATION FORCE BALANCE MEASUREMENTS

Measuring radiation force in conjunction with a hologram allows an independent determination of the magnitude of the hydrophone sensitivity. Indeed, Eqs. (11) and (13) provide the following expression for the absolute value of the hydrophone sensitivity:

$$|M| = \sqrt{\frac{1}{8\pi^2 \rho c^2 F_z} \iint\limits_{k_x^2+k_y^2 \le k^2} \left(1 - \frac{k_x^2 + k_y^2}{k^2}\right) \left| \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} dxdy \, U(x, y) e^{-ik_x x - ik_y y} \right|^2 dk_x dk_y} \tag{20}$$

The same relation can be also expressed from Eq. (16) in the following way:

$$|M| = \sqrt{\frac{1}{2\rho c^2 F_z} \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} dxdy \left\{ |U|^2 - \frac{|\partial U/\partial x|^2 + |\partial U/\partial y|^2}{k^2} \right\}} \tag{21}$$

In the current study, a PVDF hydrophone (SEA PVDF-GL-0150-1A, USA) was calibrated based on the results of acoustic holography measurements (see Figure 1) and radiation force balance measurements. Corresponding radiation force measurements were made using a special absorber (14mm thick HAM A Absorber, Precision Acoustics LTD, UK) placed on a precision balance (Precisa 310 C). From Eq. (21), these measurements provided an estimate of the hydrophone sensitivity as $|M| = 0.246\ \mathrm{V/MPa}$.

DISCUSSION

Equations (20) and (21) suggest an improved approach for single-frequency hydrophone calibration that benefits from the inherent accuracy of mass balances. It is seen from Eqs. (20) and (21) that $|M| \sim F_z^{-1/2}$, which means that the relative accuracy of calculating $|M|$ approximately scales with one half of the accuracy of the radiation force measurement. Typically commercial power balances can measure radiation force with an uncertainty of ±5%, and an uncertainty of less than 1% can be achieved for intermediate powers levels [8, 9]. While uncertainties in the structure of the acoustic field represented by a measured hologram have not been rigorously analyzed, errors in carefully measured holograms typically remain small and are associated with variations in sound speed and imperfect directionality of the hydrophone. If the hologram uncertainty is indeed small or negligible with regard to the corresponding angular spectrum, the approach described here would permit single-frequency hydrophone calibrations with an accuracy that scales with the square root of the force balance uncertainty. Consequently, the combination of force-balance and holography measurements may be capable of yielding hydrophone calibrations in the megahertz range with an accuracy comparable to that of the best existing methods [10].

ACKNOWLEDGMENTS

We acknowledge funding from RFBR 11-02-01189 and 12-02-31925, and NIH EB007643.

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