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METHOD OF MEASUREMENT OF VIBRATIONAL VELOCITY ON ULTRASONIC SOURCE SURFACE: NUMERICAL ANALYSIS OF ACCURACY

A.V. Morozov, O.A. Sapozhnikov, and Yu.A. Pishchalnikov
Lomonosov State University, Vorobyovy Gory, Moscow 119992, Russia
(e-mail: yura@acs366.phys.msu.ru)

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We have numerically analyzed the earlier calculation method for vibration velocity distribution on the surface of an ultrasound transducer, revealed possible errors of the technique, and determined the optimal operational parameters. We have also investigated the dependence of calculation accuracy on possible experimental errors, and developed recommendations on how to select operational parameters in order to improve the precision.

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1. Introduction

Ultrasound in liquids is conventionally generated by piezoelectric-plate vibrators. The radiated acoustic field is most usually calculated by an approximate method using the Rayleigh integral being a mathematical expression of the Huygens–Fresnel principle [1]. According to the latter, each point of the radiating surface is considered a point source of secondary waves. The acoustic pressure in a given point is expressed as a superposition of spherical waves with amplitudes proportional to normal velocity components at the corresponding points of the radiating surface. However, thickness fluctuations in piezoelectric plates are accompanied by other hard-to-control modes, in particular, Lamb waves [2, 3]. As a result, the distribution of the normal velocity along the source surface is actually unknown, and accurate calculation of the source fields becomes impossible. This makes it important to develop new methods to find the nature of piezoplate surface vibrations. There are several possible solutions to this problem.

As for source surface vibrations in air, their vibrational velocity is conventionally directly measured using an optical interferometer. This method provides high spatiotemporal resolution. However, such measurements in a liquid are complicated by strong acousto-optical interaction [2].

There are indirect methods for the calculation of the vibrational velocity of particles in space by the measured distribution of the acoustic pressure in a plane. One of them is the angular spectrum method allowing one to calculate acoustic field propagation between parallel planes. Currently, this technique also allows one to take into account nonlinearity, absorption, dispersion, refraction, and phase distortion effects [4]. A disadvantage of this technique is the necessity of field measurements and calculations at plane surfaces. In addition, the field reconstruction at each source point requires double calculation of the double integral, which complicates the calculation and increases the time it takes.

The equivalent phased array method considered in [5] is as follows. The source is discretized into a great number (N) of elements; in other words, it is replaced by a phased array, and the Rayleigh integral is written as the matrix product $P = HV$. To determine the complex amplitudes at the sources (column V), one is to measure the pressure (column P) at not fewer than N points of a plane arranged in front of the source. The method requires the highest accuracy of pressure measurements and is extremely dependent on noise and shifts [5]. In addition, numerical inversion of matrix H takes significant time (up to 10 h).

In the method of superposition of Gaussian beams, the wave is represented as a sum of basis Gaussian beams with different radii. The central problem is to determine their parameters. The advantage of this method is the possibility it offers to approximate the fields by very few basis beams, which significantly simplifies the procedure of field estimation in ultrasonic diagnostics problems. However, the method is restricted to axially symmetric fields and may feature significant disagreements to the experiment in the near field.

The wave phase conjugation (WPC) method (discussed in [6]) is based on the invariance of the wave equation with respect to the time reversal (in an inhomogeneous medium without absorption). For any field $P(\mathbf{r}, t)$, there is counterpropagating field $P(\mathbf{r}, -t)$ collected at sources as if the time ran backward. If the field is measured on a closed surface surrounding the sources and then reradiated with WPC, the waves will propagate towards the source and reconstruct the initial distribution. In the experiment, the sources may be surrounded by an array of sources, measure and memorize the temporal dependences of the wave amplitude and phase; then the field may be reradiated with time reversal. In practice, several hundred elements arranged in a bounded region are used. Currently, the WPC methods using the medium natural nonlinearity are also employed.

The back propagation method developed in [7–10] and used in this study differs from the WPC method by the fact that the wave phase is conjugated numerically, rather than by reradiation. First, the field distribution is measured on a plane arranged in front of the source (conventionally, using a hydrophone, over a large number of points). Then the wave phase is numerically conjugated and used to calculate the field distribution over the source. In this work, we theoretically study and experimentally check the potentials of this method by analyzing possible errors and searching for the optimal parameters of the problem. We also study the dependence of the reconstruction accuracy on possible experimental errors.

2. Description of the method

We consider a spherical source placed on the surface Σ_1 (see Fig. 1). The acoustic perturbations radiated by it propagate mostly to the right. We also consider a closed volume bounded by the radiating surface Σ_1, the plane region Σ_2, and the lateral surface $\Delta\Sigma$. The concept is as follows. Let the region Σ_2 have much larger diameter than the source. Then, placing a mirror to conjugate the wave phase instead of plane Σ_2, we collect the reflected wave at the source, thus reconstructing the field. This wave property follows from the fact that the wave equation is invariant with respect to the time reversal operation $t \to -t$. Strictly speaking, the result will be exact if the mirror conjugating the wave phase is placed not only in plane Σ_2, but also on lateral surface $\Delta\Sigma$ and on source surface Σ_1 itself. However, the contribution of $\Delta\Sigma$ can be neglected if the solid angle encompassing the source is small. The contribution from surface Σ_1 is small if the surface is close to the plane. Therefore, the acoustic pressure measurements in plane Σ_2 may be considered sufficient to reconstruct the field at the source. The corresponding equations are given in [7].

As an example, we estimate the accuracy of the method and the selection of optimal parameters for the radiator used in [7–10]: the diameter is $D = 10$ cm, the focal length is $F = 20$ cm, the generated signal frequency is $f = 1$ MHz, and the velocity of sound in water is $c_0 = 1490$ m·s^{-1}.

We will simplify the notions used as follows: "velocity" is the reconstructed distribution of the complex amplitude of the normal component of the vibrational velocity vector on the ultrasonic source surface, "pressure" is the calculated or measured

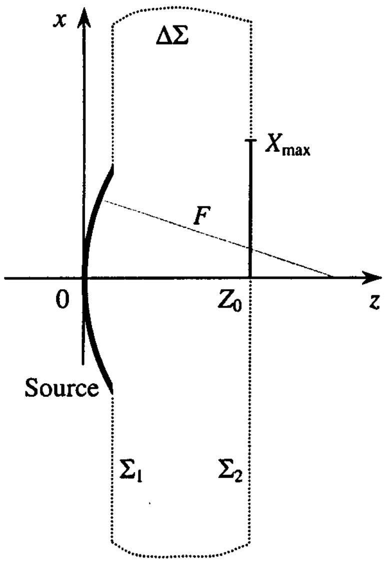

Figure 1. Geometry of the problem.

distribution of the acoustic pressure complex amplitude over the reference plane, and "reference plane" is an auxiliary area of the plane perpendicular to the acoustic axis of the ultrasonic source. In this area (singled out in Fig. 1) of plane Σ_2, the pressure is measured or calculated.

To calculate the velocity reconstruction errors, we created multipurpose Delphi code designed to (i) calculate the pressure distribution over the reference plane by a given velocity distribution on the source surface, (ii) calculate the velocity distribution on the spherical source surface by a given pressure distribution over the reference plane, (iii) set various parameters of the problem, (iv) calculate the velocity measuring errors, (v) enter errors into the problem parameters, and (vi) study the variations of the velocity reconstruction results. In the calculations, the source was supposed to be axisymmetrical; that is, the velocity at the source and the field generated by the source depended only on the distance to the axis $0z$.

3. Search for the optimum parameters of the problem

To reconstruct the vibrational velocity distribution on the source surface by a known pressure distribution as accurately as possible, the measurements and calculations are to be carried out at optimum parameters of the problem. To find these parameters, the error was minimized by three parameters: (i) the number of points N_p over the transverse coordinate along the reference plane, at which the acoustic pressure is calculated, (ii) the reference plane position Z_0, and (iii) its size X_{max}.

The accuracy was studied by comparing a reference velocity distribution (piston distribution, at which the velocity is constant along the source surfaces and is zero outside of it) with the distribution calculated by the code. First, the pressure on the reference plane was numerically calculated using the Rayleigh integral on the basis of the piston distribution. Then the pressure distribution found was used to reconstruct the velocity at the source, which differed from the initial piston distribution by the error of the method. The reconstruction accuracy was characterized by the root-mean-square deviation σ of the calculated velocity amplitude from the initial piston amplitude. Since the velocity drastically changes at the source edge due to initial piston distribution, a small vicinity (10%) of the source rim was excluded from the error calculation.

In the study of the dependence of σ on X_{max}, the reference plane subinterval was taken rather small, $H = X_{max}/N_p = 0.05$ cm, in order to exclude the corresponding error. Small X_{max} resulted in significant distortions in the reconstruction of the initial piston velocity distribution, since only the paraxial region of the field was included in the calculation. As the calculation showed, the best reconstruction accuracy was reached at $X_{max} \simeq 10$ cm. The error σ was virtually independent of X_{max} as it further increased. To decrease the number N_p of measurement points, we took $X_{max} = 10$ cm. The calculation showed that the diameter of the measurement region is to be in general larger than the beam cross size by an order of magnitude in order to cover the whole field.

When the whole beam is covered (rather large X_{max}), the reconstruction can be inaccurate because of an insufficiently short measuring step H (insufficient number N_p of points of division in the refer-

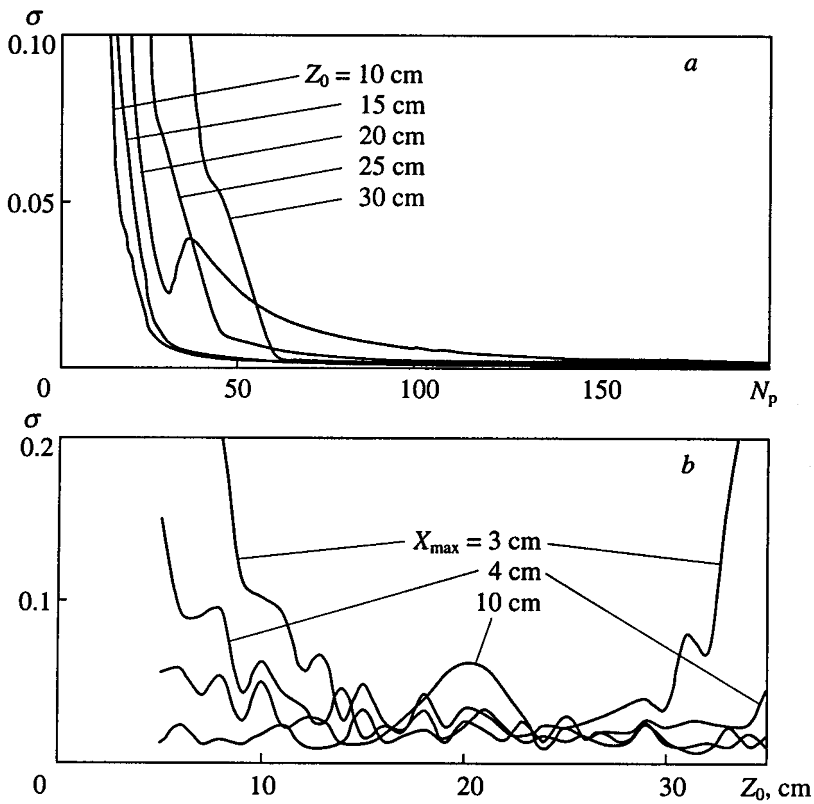

Figure 2. Dependence of the calculation errors σ on problem parameters N_p and Z_0.

ence plane). Figure 2*a* shows the dependence of the error σ on N_p at $X_{max} = 10$ cm and various Z_0. One can see that the measurement accuracy is the best at $N_p > 100$. Therefore, $N_p = 100$ can be used as an optimum parameter.

At given X_{max} and N_p, the reconstruction accuracy also depends on the position of the reference plane. This is illustrated in Fig. 2*b*. The figure shows the dependence of the error σ on the reference plane position Z_0 at $N_p = 100$ and various X_{max}. As the reference plane is placed near the source, the error increases, since the field distribution is wide and strongly irregular: the characteristic spatial scale of the wave amplitude nonuniformity is of the order of $\lambda Z_0/2D$, where λ is the wavelength. Therefore, a larger measurement window and a short step, hence many points, are necessary. This makes measurements near the source inconvenient. Measurements far beyond the focus are also inexpedient, since the wave parameters can be distorted there by absorption and acoustic nonlinearity of the medium.

In the focus itself, the distribution is very narrow; therefore, a very short step is required to provide good reconstruction accuracy. If the hydrophone size and the positioning system step are sufficiently small, the reconstruction by the field measured in the focus yields good accuracy. An analysis showed the focus vicinity to indeed feature smaller errors at the setup parameters we used; however, the optimum position of the reference plane corresponds to the prefocal region, rather than the focus itself.

One can see in Fig. 2*b* that the measurement accuracy is the best at $Z_0 = 16$ cm with σ virtually independent of X_{max} at this point. Therefore, we accepted $Z_0 = 16$ cm to be the last optimum parameter. Thus, the optimum parameters for the case under consideration were found to be as follows: $N_p = 100$, $X_{max} = 10$ cm, and $Z_0 = 16$ cm. The reconstruction error did not exceed 1 %.

Thus, the velocity reconstruction accuracy substantially depends on many parameters; therefore, numerical simulation is to be carried out before measurements.

Our accuracy measurement was carried out in a one-dimensional configuration with axial field symmetry. In the case of two-dimensional configuration, the dependence of the reconstruction accuracy on various parameters was not so thoroughly studied, since the calculations take time longer by an order of magnitude. However, the general conclusions remain valid. Two-dimensional calculation can also provide sufficiently high accuracy of the velocity reconstruction. For example, at $Z_0 =$ 15 cm, $X_{max} = 3$ cm, and 40×40 points over the pressure and velocity, the error estimated by our technique did not exceed 2 %.

4. Impact of experimental errors on the velocity reconstruction accuracy

In practical experiments, the factors influencing the velocity reconstruction accuracy are not only the selection of N_p, X_{max}, and Z_0, but also pressure measurement errors (receiving channel noise), inaccurate knowledge of the reference plane position and inclination angle, and the velocity of sound in the medium. To reveal the impact of these errors upon the reconstruction accuracy, a numerical study was carried out. As in the search for the optimum parameters, first the pressure complex amplitude P_0 was calculated from the piston velocity distribution, using the reference plane position Z_0 and the velocity of sound c_0 in the medium. Then, the pressure distribution determined was used to reconstruct the velocity at the source, but now using the inaccurate parameters: $Z_0 + dZ_0$, $c_0 + dc_0$, and $P_0 + dP_0$, where dZ_0, dc_0, and dP_0 are artificially introduced small perturbations. The velocity calculated in this way was compared to the piston distribution; thus, the reconstruction error σ was determined. The calculation involved the optimum problem parameters previously obtained.

The study showed that when the reference plane position is inaccurately specified, the size of the reconstructed velocity distribution, that is, the visible source size, increases at $dZ_0 > 0$ and decreases at $dZ_0 < 0$. In this case, the distribution amplitude decreases at $dZ_0 > 0$ and increases at $dZ_0 < 0$. The phase of the reconstructed velocity also appreciably changes. Similar results were obtained when the velocity of sound in the medium was set incorrectly.

The pressure measurement accuracy is affected by noise of the receiving channel (hydrophone, preamplifier, coaxial cable, and oscilloscope). Another factor is the finite size of the sensitive region of a needle hydrophone. Thus, the code reconstructs the velocity distribution over the source by the inaccurate pressure distribution. To study the impact of noise of the receiving channel, small random complex additions with a distribution function homogeneous over the given interval were introduced into the numerically calculated pressure distribution. The numerical experiment showed that noise of the receiving channel have a significant effect on the velocity reconstruction accuracy. If the noise effect is neglected, it is expedient to make the pressure measurement region sufficiently large in order to cover the whole field. The noise changes the situation, since the signal-to-noise ratio at the beam edges becomes low at an excessive increase of the measurement region size X_{max}, and these regions introduce significant distortions during the reconstruction. The optimum size X_{max} of the region of pressure measurement may be selected by numerical simulation.

In pressure measurements, it is important to know the inclination angle of the reference plane in order to correctly reconstruct the velocity. The impact of the inclination angle error is illustrated in Fig. 3 showing the results of measurement. The experimental setup is described in [1]. The pressure was measured (see Fig. 3*a*) in the reference plane, the normal to which was inclined by 4° with respect to the acoustic beam axis. To make this error in the experiment is very easy. Then the velocity at the source was reconstructed, supposing the reference plane was not inclined. The distribution amplitude remained symmetric, but was slightly

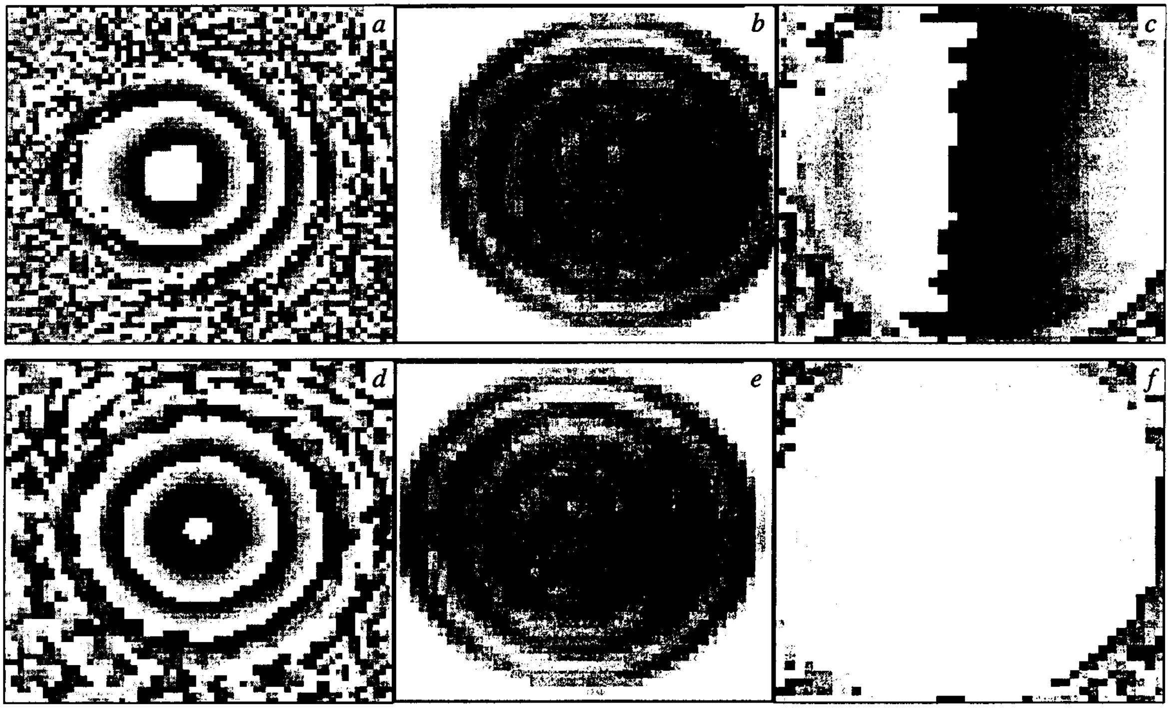

Figure 3. Pressure measured at the inclined plane and the velocity calculated at the source (see text).

displaced to the right (Fig. 3*b*). The wave front was also appreciably inclined (Fig. 3*c*). Using the code for velocity calculation, the inclination angle of the measurement plane can be selected so that the reconstructed velocity phase acquire spherical symmetry (Fig. 3*f*), and the amplitude distribution be shifted towards the center (Fig. 3*e*). Thus we determine the true angle of the reference plane inclination to the source. This technique allows one to find the inclination angle to the accuracy of 0.1°, hence compensate for the error caused by setting the inclination angle of the measurement plane.

5. Conclusion

The accuracy of the method of the vibrational velocity reconstruction at the acoustic radiator surface has been numerically and theoretically studied. The reconstruction accuracy was shown to substantially depend on a variety of parameters; therefore, numerical simulation is to be carried out before measurements. The impact of noise of the receiving channel, various experimental errors, errors caused by setting the velocity of sound in the medium, and the errors in the position and the inclination angle of the pressure measurement plane have been studied. We give certain recommendations on the selection of the problem parameters with the purpose to diminish experimental errors. The method has been shown to be highly accurate in velocity reconstruction at corresponding parameters and reasonable measurement time.

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