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NONLINEAR ACOUSTICS

Mechanisms for Saturation of Nonlinear Pulsed and Periodic Signals in Focused Acoustic Beams

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Abstract—Acoustic fields of powerful ultrasound sources with Gaussian spatial apodization and initial excitation in the form of a periodic wave or single pulse are examined based on the numerical solution of the Khokhlov–Zabolotskaya–Kuznetsov equation. The influence of nonlinear effects on the spatial structure of focused beams, as well as on the limiting values of the acoustic field parameters is compared. It is demonstrated that pressure saturation in periodic fields is mainly due to the effect of nonlinear absorption at a shock front, while in pulsed fields is due to the effect of nonlinear refraction. The limiting attainable values for the peak positive pressure in periodic fields turned out to be higher than the analogous values in pulsed acoustic fields. The total energy in a beam of periodic waves decreases with the distance from the source faster than in the case of a pulsed field, but it becomes concentrated within much smaller spatial region in the vicinity of the focus. These special features of nonlinear effect manifestation provide an opportunity to use pulsed beams for more efficient delivery of wave energy to the focus and to use periodic beams for attaining higher values of pressure in the focal region.

Keywords: ultrasound, periodic and pulsed signals, nonlinear effects, focused beams, nonlinear absorption, nonlinear refraction.

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INTRODUCTION

Investigation of focusing for high-amplitude pulsed and periodic signals is an important trend in nonlinear acoustics [1]. The interest to this topic is connected in particular with various medical applications of powerful ultrasound. Focused pulsed shock-wave beams are used, for example, in lithotripsy [2] for destruction of kidney stones and periodic waves are used in noninvasive surgery for destruction of soft tissue tumors [3, 4]. The efficiency of these procedures depends strongly on the operational mode of a radiator, i.e., the number and shapes of generated pulses and their amplitude and length. To select the most optimal mode of radiator operation it is necessary to be able to predict the parameters of created fields and the biological effects caused by them. It is necessary also to take into account the that the wave amplitude growth in the process of focusing is accompanied by strengthening of nonlinear effects and leads to formation of shock fronts in the focal region of a beam. In the case of strong manifestation of nonlinear effects, the effect of saturation is observed: the acoustic field parameters in the radiator focus stop depending on the pressure amplitude at the source.

The mechanisms causing saturation are different for periodic and pulsed fields. This leads to the fact that the limiting values for the parameters of an acoustic field in the focus also differ for periodic and pulsed modes of focusing. For example, frequently in medical applications it is necessary to obtain a high value for the peak positive or negative pressure in the radiator focus. In a weak-nonlinear case, it is sufficient for this to increase pressure amplitude at the source. However, if nonlinear effects in a medium are considerable, the pressure increase at the source does not provide pressure increase in the radiator focus because of the saturation effect. To attain higher pressure amplitude in the focus, in this case it is possible to use a signal with a different time structure at the source. Here we compare the efficiencies of focusing for pulsed and periodic wave beams, as well as physical mechanisms causing saturation in the fields under investigation.

The phenomena of limiting the pressure value in the focuses of periodic and pulsed beams are described in reviews [5, 6]. The effects of saturation in periodic fields generated by piston focused sources were investigated in detail both analytically in the approximation of geometrical acoustics [7] and numerically taking into account nonlinear and diffraction effects [8]. To describe nonlinear sound beams in a preshock mode, the Gaussian beam model was used [9]. It is possible to make analytical calculation much simpler within this model. A model for a powerful source with apodization close to the Gaussian one is also interesting from the point of view of practice [2]. It is necessary to note that the saturation effects for focused sources with the Gaussian spatial apodization have been investigated much more poorly than in the case of piston sources. The results obtained in the approximation of geometrical acoustics, i.e., not taking into account diffraction effects, are known. For example, in [10], to describe nonlinear effects in a focused beam, the propagation of a wave in a ray tube with a cross section area varying with distance in the same manner as the cross section of a linear Gaussian beam considered. A similar approach provided an opportunity to estimate analytically the saturation pressure for a periodic acoustic field. Apart from periodic fields, the pulsed signals used in lithotripsy are also interesting [2]. In [11] estimates for the limiting value of pressure in the radiator focus and the dimensions of the focal region were obtained proceeding from analysis of focusing for a pulsed Gaussian beam under the assumption of absence of diffraction. However, the obtained estimates are approximate. To obtain more precise quantitative results, it is necessary to take into account also the diffraction and thermoviscous absorption effects, in addition to the nonlinear ones. The equations taking into account the above effects have no analytical solutions and, therefore, need a numerical analysis.

Numerical simulation of focusing for shock pulses is a more complex problem in comparison with simulation of periodic waves, since it needs more calculation resources. In particular, in the case of identical steps for the time and spatial grids for simulating focusing of pulsed signals, it is necessary to use time windows with a larger dimension (more than ten times for weakly focused beams). This leads to an increase of the random-access memory needed for calculation and a corresponding increase of calculation time even in the case of linear propagation. Numerical description of pulse focusing in a nonlinear mode became possible only lately due to the rapid progress of computing machinery and methods of parallel computing.

Here, using the Khokhlov–Zabolotskaya–Kuznetsov equation, we simulate periodic and pulsed acoustic fields generated by focused sources with Gaussian spatial apodization. The waveform at the source is selected in the form of a single pulse or a harmonic wave. Waveforms in the radiator focus are calculated, the point where the maximum peak positive pressure of an acoustic field is attained being taken as the focus. Two-dimensional spatial distributions of peak positive and negative pressures for periodic and pulsed fields, saturation curves, and the dependences of the beam energy on the distance from the source are compared. The results are obtained for different values of the linear gain factor and pressure amplitude at the source including the values used in practice [2]. The obtained numerical solutions are compared with each other and known analytical estimates.

NUMERICAL MODEL

Nonlinear propagation of focused acoustic waves can be described using the Khokhlov–Zabolotskaya–Kuznetsov equation taking into account nonlinear effects, diffraction, and absorption. The equation in dimensionless variables for axially symmetrical beams has the form

$$\frac{\partial}{\partial\theta}\left[\frac{\partial P}{\partial\sigma} - NP\frac{\partial P}{\partial\theta} - A\frac{\partial^2 P}{\partial\theta^2}\right] = \frac{1}{4G}\left(\frac{\partial^2 P}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial P}{\partial\rho}\right), \quad (1)$$

where $P = p/p_0$ is the acoustic pressure normalized to the initial wave amplitude p_0 at the source, $\sigma = x/F$ is the coordinate along the beam axis that is normalized to the focal distance F, $\rho = r/a_0$ is the transverse coordinate normalized to the source radius a_0, $\theta = 2\pi\tau/T_0$ is the dimensionless time, $\tau = t - x/c_0$ is the time in the traveling coordinate system, c_0 is the sound velocity in water, and T_0 is the pulse length (in the case of a harmonic wave, it is the length of its single period). Equation (1) contains three dimensionless parameters, i.e., the nonlinearity parameter $N = 2\pi F\varepsilon p_0/\rho_0 c_0^3 T_0$, where ε is the nonlinearity coefficient of a medium and ρ_0 is the density of a medium, the diffraction parameter $G = \pi a_0^2/c_0 F T_0$, and the absorption parameter A.

The initial conditions were selected so that the waveforms in the focus would be identical for pulsed and periodic fields in the case of linear focusing and the identical values of the peak positive pressure P_+ (Fig. 1) would be attained. A harmonic wave was selected as the initial periodic signal,

$$P_0(\theta) = \sin\theta. \quad (2)$$

A pulsed mode was simulated by a periodic sequence of pulses with a large off-duty ratio in the form of a single period of a harmonic wave and the pressure between pulses was taken to be constant. In this case the signal value average over the time window was zero,

$$P_0(\theta) = \begin{cases} 1 - 1/n_0 - \sin\theta, & \pi/2 \le \theta \le 5\pi/2, \\ -1/n_0, & \theta \le \pi/2 \text{ and } \theta \ge 5\pi/2, \end{cases} \quad (3)$$

where $2\pi n_0$ is the length of the time window and n_0 is an integer number (in the case of $G = 10$, the value $n_0 = 13$ was selected). The choice of the initial pulse profile in the form of a single period of a harmonic wave was convenient for further comparison of focusing for periodic and pulsed fields in a nonlinear case.

The boundary condition was set in the plane $\sigma = 0$ and corresponded to a circular focused source with Gaussian apodization. Beam focusing was provided by introduction of a phase shift increasing quadratically with the transverse coordinate,

$$P(\sigma = 0, \rho, \theta) = P_0\left(\theta + G\rho^2\right)\exp(-\rho^2). \quad (4)$$

A method of splitting in physical factors and a frequency–time approach were used to calculate various effects in the numerical algorithm for solving Eq. (1) with the boundary condition of Eq. (4) and the initial time profiles (Eqs. (2) and (3)) at each step of a grid in the coordinate s. Diffraction effects were calculated in a spectral representation with the help of the Crank–Nicholson scheme of the second order of accuracy over both spatial coordinates [8]. Taking into account dissipative effects was performed also in a spectral representation using an exact solution for each harmonic [12]. Nonlinear effects were calculated in a time representation. To do this a conservative Godunov's type numerical algorithm of the second order of accuracy in time and the first order of accuracy in the propagation coordinate was used [13]. This algorithm provides an opportunity to describe propagation of nonlinear waves with shock fronts even in the case in which only three or four nodes of a time grid fit a wave shock front. Transition between the spectral and time representations was performed with the help of the fast Fourier transformation. The algorithm was adapted for parallel computation with the help of the OPENMP technology, which provided an opportunity to reduce significantly the calculation time. To simulate a focused beam numerically, we selected the following values of the parameters in Eq. (1): $G = 10$, 20, and 40 and $0 \leq N \leq 6$. The following physical parameters correspond to real electric-discharge sources used in lithotripsy: the effective reflector radius $a_0 = 77$ mm, the effective focal distance $F = 128$ mm, the pressure amplitude $p_0 = 6$ MPa, and the radiated pulse length $T_0 = 4$ μs (a Dornier HM3 lithotripter [2]). The values of dimensionless parameters in Eq. (1) in this case are: $G = 14$ and $N = 1.4$. Here the major results of numerical calculation will be presented for the values $G = 10$ and $N = 1.0$ that are close to real values used in lithotripters. In the case of periodic fields, the above values are also typical, for example, in the problems of therapy for soft tissues with the use of medical ultrasonic transducers [3].

The parameters for the numerical scheme were selected based on the condition of stability for a numerical algorithm and a preset accuracy of calculation (2%). The calculation accuracy was estimated by comparing the solutions obtained at varying the discretization step two times. If in this case the solutions differed by less than 2%, the discretization step was taken to be equal to the current one. The diffraction step along the propagation coordinate was $h_\sigma = 10^{-3}$ and the step in the transverse coordinate $h_\rho = 4 \times 10^{-4}$. To satisfy the Kurant–Friedrichs–Levi condition in the Godunov's type scheme for a nonlinear operator several steps over nonlinearity were performed within each diffraction step along the propagation coordinate. The size of the nonlinearity steps h_{σ_nonl} was selected automatically at each diffraction step h_σ and varied within the limits $7 \times 10^{-5} \leq h_{\sigma\_nonl} \leq 3 \times 10^{-4}$. The step of time discretization or the number of the harmonics taken into account in calculation also varied with wave propagation. The initial number of harmonics in the case of periodic wave focusing was $n = 128$, and in the case of pulse focusing it was $n = 128n_0$. This number of harmonics was sufficient to describe wave focusing with the selected precision in linear and weakly nonlinear cases ($N < 0.1$). As a wave propagated nonlinearly and, correspondingly, the wave front became steeper, the number of the harmonics taken into account increased. In the focal region, in the case of calculating the propagation of shock fronts, we used 2048 harmonics for a periodic wave and 8192 harmonics in the case of a pulsed signal. Thus the minimum time step for a pulsed field was $h_\theta = 5 \times 10^{-3}$ and $h_\theta = 1.5 \times 10^{-3}$ for a periodic field. Artificial absorption was introduced for smoothing large field gradients in transverse directions, which leads to algorithm divergence [14]. Its value was selected from the condition for at least three nodes of the time grid to fit a shock front in the focus. In this case near the source the absorption was small and increased as the distance to the geometrical focus of the source became smaller. In the focal region of a beam, the artificial absorption increased up to tenfold at $G = 10$ and

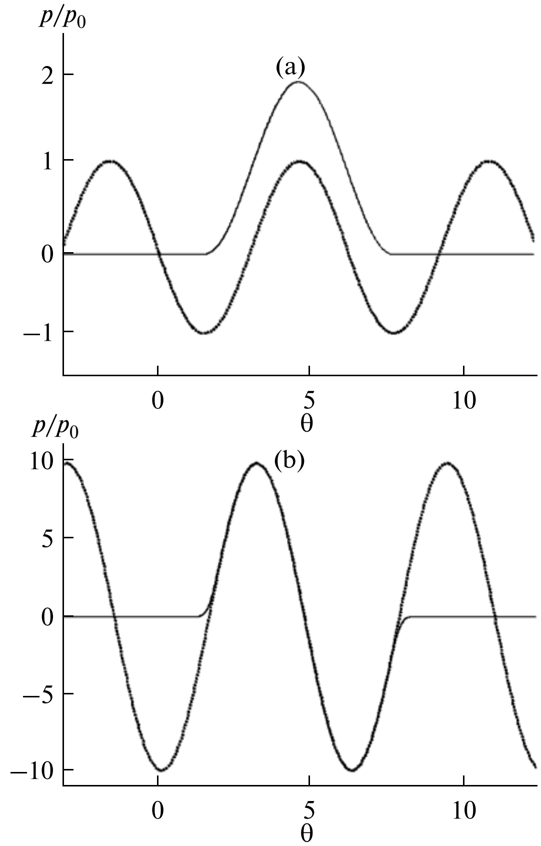

Fig. 1. Waveforms (a) at the radiator and (b) in the radiator focus in the case of linear focusing with the concentration coefficient $G = 10$. A solid line corresponds to a pulsed signal; a dotted line, to a periodic wave.

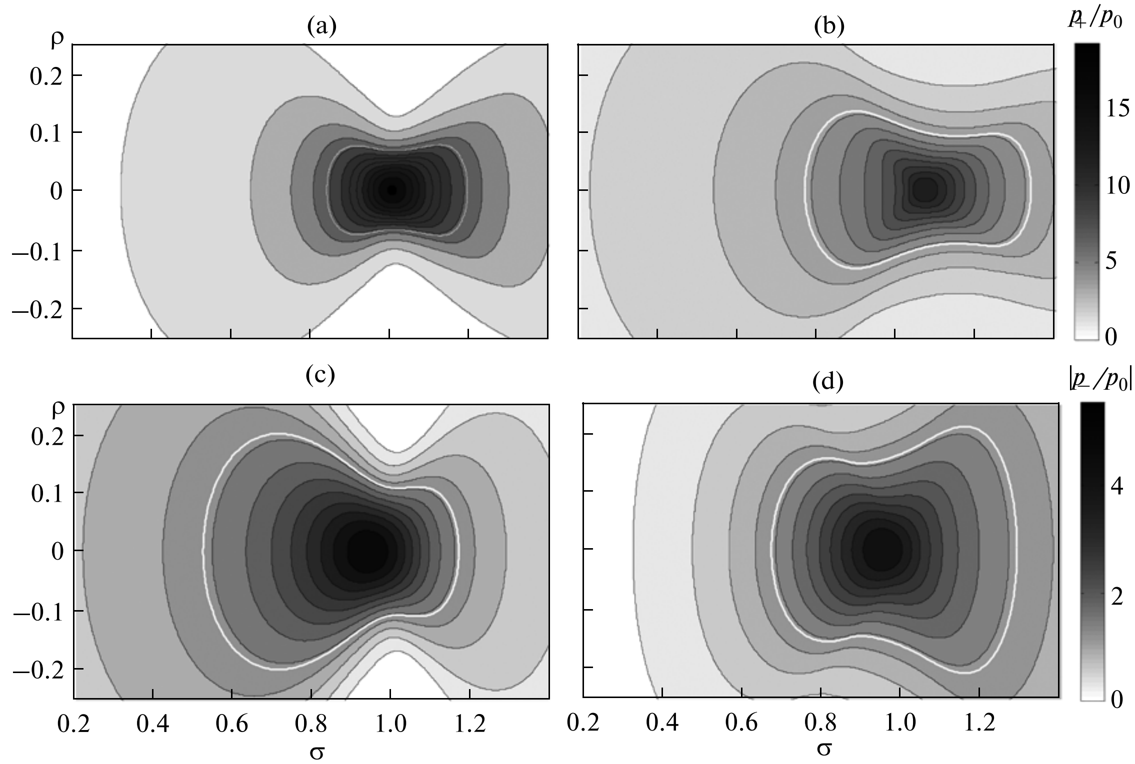

Fig. 2. Two-dimensional spatial distributions for the peak (a, b) positive and (c, d) negative pressures in (a, c) periodic and (b, d) pulsed focused fields ($G = 10$ and $N = 1.0$). White contours show the boundaries of focal regions. Shades of gray indicate the levels of the peak (a, b) positive and (c, d) negative pressures.

40 times at $G = 40$. The minimum value of the absorption coefficient was $A = 5.4 \times 10^{-3}$.

In the process of problem simulation, we used a priori knowledge on the focused geometry of a beam. For example, in the vicinity of the focus, the calculation in the transverse coordinate was conducted only over the region where the peak positive pressure exceeded 0.06% of its maximum value.

RESULTS

Figure 2 presents two-dimensional patterns of spatial distributions for the peak (a, b) positive and (c, d) negative pressures in a nonlinear beam ($G = 10$ and $N = 1.0$) for (a, c) periodic and (b, d) pulsed fields. The distance $\sigma = 1$ corresponds to the geometrical focus of a source (the source is on the left at $\sigma = 0$). One can see clearly that larger values of both peak positive and negative pressures are attained in a periodic field in comparison with the corresponding values in a pulsed field. At the same time, the focal region of the peak positive pressure in a periodic field is more compact in both the longitudinal and transverse directions. In Fig. 2 the boundaries of the focal region are determined at the level e^{-1} of the maximum pressure in each field and indicated by a white contour. Thus, if the purpose of focusing is attaining high values of the peak positive and negative pressures, it is preferable to use not pulsed, but periodic, focused beams.

To describe cavitation effects, it is important to know the distribution of the peak negative pressure P_-. As one can see from Fig. 2, the focal spot for P_- has a nonsymmetrical shape in the form of a large-dimension nonsymmetrical dumbbell in the direction of the source for a periodic field (Fig. 2c) and in the opposite direction for a pulsed field (Fig. 2d). This difference is connected with the fact that there is no rarefaction phase in the initial pulse profile. It appears only far from the source because of manifestation of diffraction effects. The center of the focal region for the peak negative pressure for both fields is shifted to the source: the maximum absolute value for the peak negative pressure is attained at $\sigma \approx 0.95$.

Figure 3 presents ray patterns for (a) periodic and (b) pulsed fields at the parameter values $G = 10$ and $N = 1.0$. A dashed line indicates wave fronts that were determined at each spatial point according to the maximum of the derivative from a waveform at this point. Solid lines indicate rays plotted as perpendicular to wave fronts in dimensional coordinates. Shades of gray show the levels of the peak positive pressure. The point of the spatial maximum for the peak positive pressure P_+ of the field is taken for the source focus. For a periodic field, the maximum of P_+ is attained approximately in the geometrical focus of the source

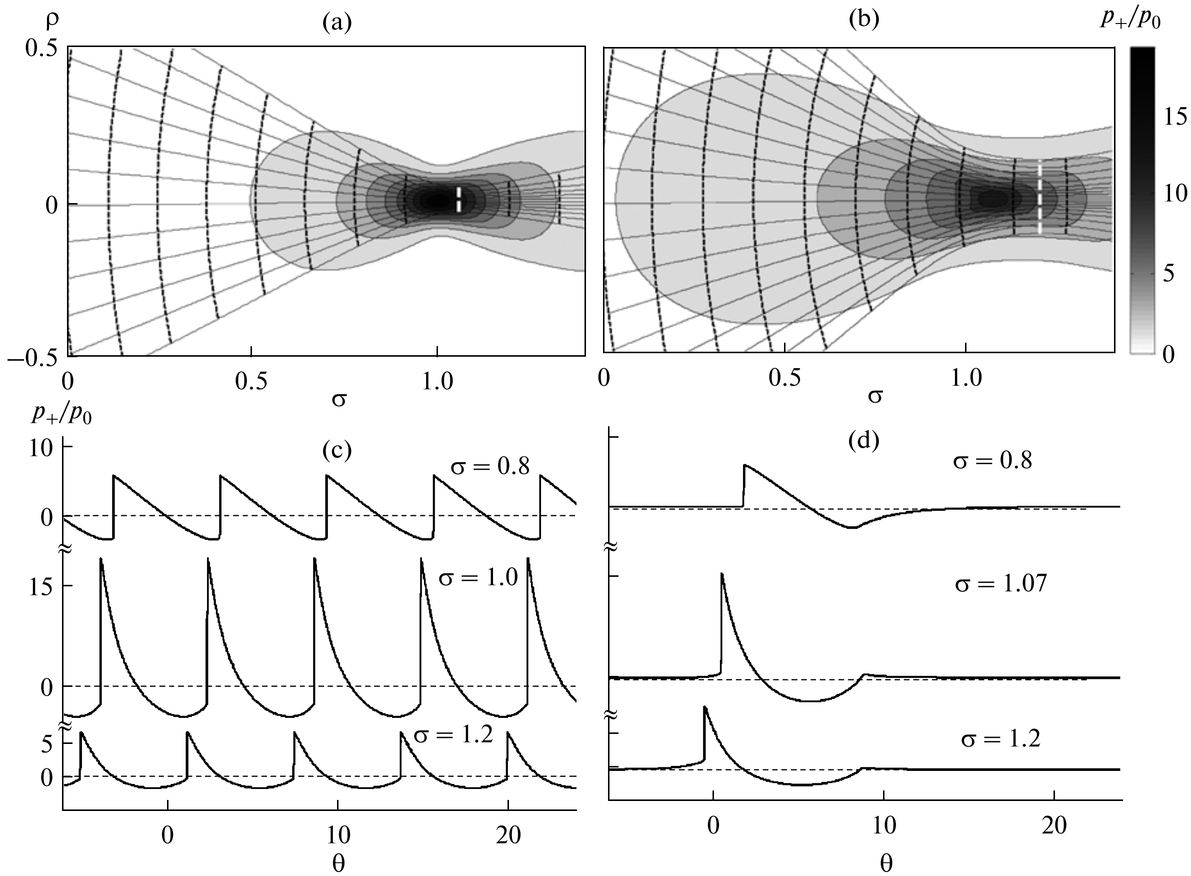

Fig. 3. The upper series. Ray patterns for (a) periodic and (b) pulsed fields. Rays are plotted by solid lines and shock fronts by broken lines. The white dashed line indicates the front position at the point of its straightening at the axis. Shades of gray show the levels of the peak positive pressure ($G = 10$ and $N = 1.0$). The lower series. Waveforms at the source axis at different distances σ in the cases of (c) periodic and (d) pulsed fields.

($\sigma = 1.0$). In pulsed fields the maximum value of pressure P_+ is attained behind the geometrical focus at $\sigma \approx 1.1$. In the linear case, the maximum of the peak positive pressure for periodic and pulsed fields is attained at $\sigma \approx 0.98$. A small shift of the focus toward the source in this case arises due to manifestation of diffraction effects [1]. Thus the focus position is different in the cases of linear and nonlinear focusing and depends on the time structure of a signal. The focus shift from the source in a nonlinear pulsed field with a nonsymmetrical shock front is caused by the phenomenon of nonlinear refraction [11]. This phenomenon arises due to the fact that the propagation velocity for the shock wave front is determined by the average value of pressure before the front and behind it. Thus, a large-amplitude shock front propagating along a nonperturbed medium has a larger velocity than the front with smaller amplitude. In the case of a source with Gaussian spatial apodization, the front velocity at the axis of a pulsed beam exceeds the front velocity at the periphery that leads to local defocusing of a beam. In periodic fields the phenomenon of nonlinear refraction manifests itself much more weakly than in pulsed fields. This is connected with the fact that the shock front formed in an initially harmonic wave stays almost symmetrical with respect to zero up to the focus. Figure 3c shows periodic waveforms at the source axis and at different distances σ from the source. One can see that, at $\sigma = 0.8$, a periodic waveform is still symmetrical with respect to zero; this means that the front velocity in the traveling coordinate system is close to zero. The pulse profile at the same value of σ (Fig. 3d) has a form such that, at the left from the shock front, pressure perturbation is zero. Therefore, the shock front of a pulse propagates with a larger velocity than does the front of a saw-tooth wave, which means that the phenomenon of nonlinear refraction manifests itself more strongly for a pulsed field. The influence of nonlinear refraction in periodic fields is significant only in the focal region, where the waveform becomes asymmetrical (a waveform at $\sigma = 1.0$). For pulsed fields the influence of nonlinear refraction becomes significant immediately after shock-front formation. The waveforms plotted at $\sigma = 1.0$ and $\sigma = 1.07$ correspond to the focuses of (c) periodic and (d) pulsed fields. As one can see from the fig-

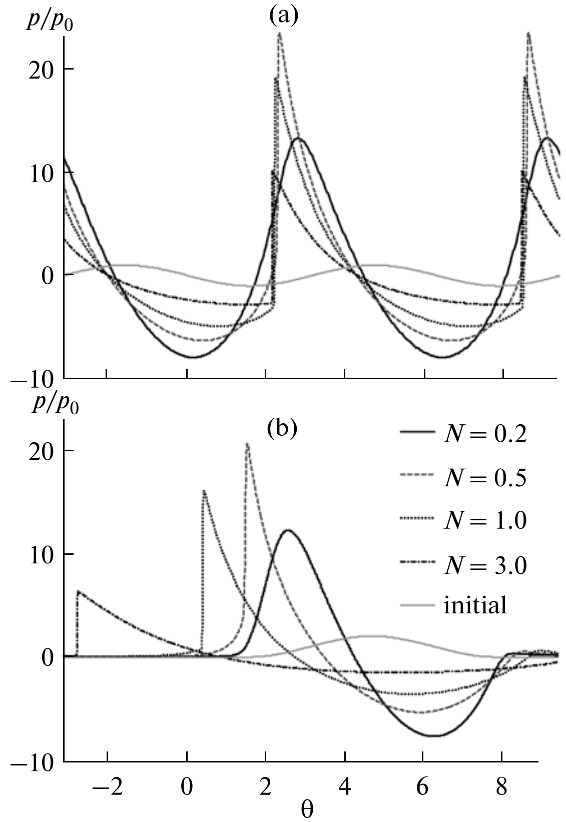

Fig. 4. Waveforms in the focus (at the point of the maximum for the peak positive pressure) for (a) periodic and (b) pulsed signals at the value of the concentration coefficient $G = 10$ and different values of the nonlinear parameter $N = 0.2$, 0.5, 1.0, and 3.0. Solid gray lines show waveforms at the source.

ure, the maximum pressure in the periodic waveform is higher than in the case of a pulse and the pulse length exceeds the length of a single period for a periodic wave. After crossing the focus (waveforms at $\sigma = 1.2$), pressure in both fields decreases and the shock front in the accompanying time coordinate system is shifted to the left (i.e., it arrives earlier than a linear wave would) because of the combined influence of nonlinear refraction and diffraction effects.

The white dashed lines in Figs. 3a and 3b indicates the wave fronts of periodic and pulsed fields at the points of their straightening at the axis. One can see clearly that straightening of a wave front occurs behind the focus, and this means that the wave front is still converging in the region, where the maximum of the peak positive pressure is attained. Wave front straightening in (b) a pulsed field occurs farther from the focus than in (a) a periodic field. This is also caused by the phenomenon of nonlinear refraction, which manifests itself much more weakly in periodic focused beams than in pulsed ones.

The characteristic distortion of waveforms in the source focus at different values of the nonlinear parameter N can be observed in Figs. 4a and 4b. The front position for a harmonic wave changes insignificantly, while the pulse front becomes strongly shifted to the left because of nonlinear refraction. As the nonlinear parameter N grows, the pulse length increases and the length of a single period for a continuous wave does not change, the initially harmonic wave becoming a saw-tooth wave. At the value for the coefficient of linear focusing $G = 10$, a singularity in a periodic wave and a pulse in the focal region starts to form at $N = 0.5$. In a weak nonlinear case (at $N < 0.5$), the value of the peak positive pressure in pulsed and periodic fields grows with the increase of the nonlinear parameter N and then decreases after formation of a shock front (at $N > 0.5$) due to absorption at the formed singularity. Before formation of a singularity (at $N < 0.5$), the compression phase of a pulse shortens and then at $N > 0.5$ it lengthens. The rarefaction phase of a pulse increases monotonically with the growth of N. In a periodic wave, the compression phase shortens as nonlinear effects become stronger and the rarefaction phase lengthens, but these changes are stressed much more weakly in comparison with the pulsed case.

Figure 5a presents saturation curves for the peak positive pressure in periodic and pulsed fields. Saturation curves for a periodic field are shown in gray and for a pulsed field in black. We assume that saturation is achieved starting from the moment at which the derivative of a saturation curve is 5% of its maximal value. In this case for a periodic field saturation of the peak positive pressure occurs at $N = 5$ and for a pulsed field, at $N = 1.5$. Thus saturation in pulsed fields occurs earlier than in periodic fields, i.e., at smaller values of the nonlinear parameter N and therefore at smaller values of pressure at the source. In a weakly nonlinear mode (at $N < 0.5$) saturation curves for periodic and pulsed fields are close to each other; i.e., the fields have close values of positive and negative pressures (Figs. 5a and 5b). At large values of the nonlinearity coefficient N, the peak positive pressure and the modulus of the peak negative pressure in a periodic field are larger than in a pulsed field. Saturation for the peak negative pressure is not observed in the interval of studied parameters N (Fig. 5b).

The following formula obtained in [11] in the approximation of nonlinear geometrical acoustics provides an opportunity to estimate the maximum value for pressure amplitude in a pulsed field:

$$P_{\max} \approx 1.5 p_{\text{in}} \alpha^2, \tag{5}$$

where $p_{\text{in}} = \dfrac{c_0^2 \rho_0}{2\varepsilon}$ is the internal pressure in a medium, α is the opening angle of the source aperture in the case of focusing, and $\alpha \approx a_0/F$. In a dimensionless form, this formula is written down as follows:

$$NP_+/G \approx 1.5. \tag{6}$$

As one can see from Fig. 5a, in the case of saturation for the peak positive pressure in pulsed fields, the numerically calculated coefficient $NP_+/G \approx 1.9$.Thus the saturation pressure in pulsed fields that is predicted according to Eq. (6) is approximately 20% smaller than that calculated in the case of more precise description, taking into account diffraction effects. It is interesting that, like in the solution to equations of geometrical acoustics (Eq. (5)), the saturation pressure turned out to be independent of the parameter G value, i.e., the initial pulse length.

An analytical expression for estimating pressure in the focus of a source for periodic waves was obtained in [10],

$$p_F = \frac{\pi p_{\mathrm{in}} \alpha^2}{\ln(2p_F/p_0)}. \tag{7}$$

Equation (7) can be written down in a dimensionless form for the peak positive pressure as

$$NP_+/G = \frac{\pi}{\ln(2G)}. \tag{8}$$

The values of saturation pressure obtained using this formula for different values of the linear focusing parameter G are shown in gray at the right from the legend in Fig. 5a. As one can see, at $N = 6$ the approximate formula of Eq. (8) produces a pressure value approximately three times smaller than that obtained in numerical calculation for a periodic field.

Since saturation curves for the peak positive pressure (Figs. 5a and 5b) in periodic and pulsed fields turned out to be almost independent of the focusing coefficient G, this provided an opportunity to select convenient approximations for saturation curves in a periodic field,

$$NP_+/G = \min\left(10N^2,\ 2.15\log(7N+1)\right), \tag{9}$$

and a pulsed field,

$$NP_+/G = \min\left(10N^2,\ 0.4\log(N-0.28)+1.64\right). \tag{10}$$

In Figs. 5a and 5b, the curves plotted using these approximations are shown by thick light-gray lines. One can see that, in both cases, the quadratic increase of the peak positive pressure in the weakly nonlinear case gives place to a slow logarithmic increase. Precisely this increase describes saturation.

An analogous approximation was also selected for estimating the limiting peak negative pressure. Since the values of the peak negative pressure in periodic and pulsed fields are close, identical approximations were selected for both fields. Opposite to the saturation curves for the peak positive pressure, the values of the peak negative pressure at the given N depend on the linear coefficient of field concentration G,

$$N|P_-|/G = \frac{\log\left[\left(\frac{G}{13}+1.2\right)N+1\right]}{\frac{G}{33}+0.61}. \tag{11}$$

Figures 6a and 6b show waveforms at different transverse distances ρ from the source axis at the distance $\sigma = 0.8$ from the source itself. One can see that,

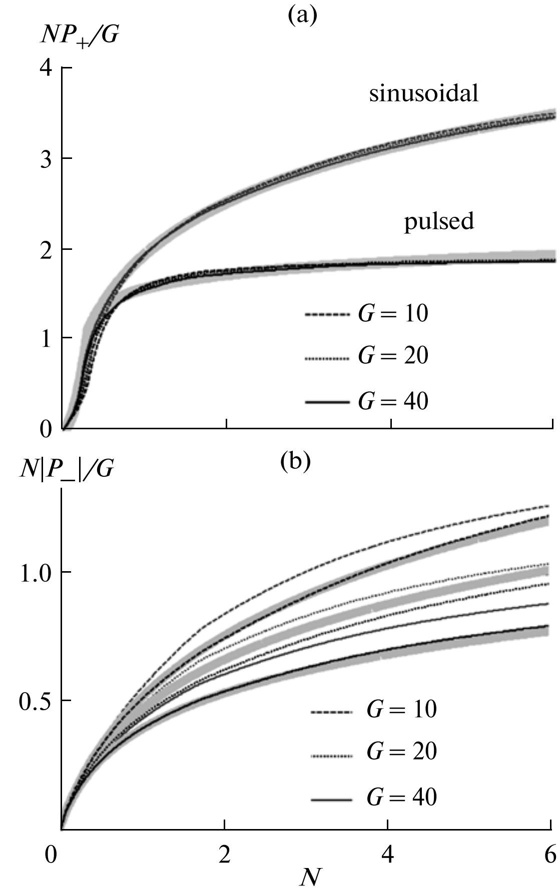

Fig. 5. Saturation curves for the peak (a) positive and (b) negative pressures. Gray color corresponds to the dependences for a periodic fields and black for a pulsed one. Thick light-gray lines show the saturation curves plotted using the approximations given by Eqs. (9)–(11). In the plot at (a) the right from the legend gray color indicates the saturation levels calculated using the analytical solution given by Eq. (8).

even at a small distance from the beam axis ($\rho = 0.24$), the waveforms are almost undistorted, though a shock front is formed at the source axis in both (a) periodic and (b) pulsed fields.

Larger values of the peak positive pressure in periodic fields (Fig. 5a) can be explained qualitatively in the following way. Because of the Gaussian spatial apodization of pressure amplitude at the source, the peak pressure in the waves coming from the central part of it is larger than from the waves coming from the source periphery. Since nonlinear effects manifest themselves stronger for waves with larger amplitudes in the cases wherein a shock front is formed in the waves from the central part of the source, the waves coming from the source periphery are distorted much weaker (Figs. 6a and 6b). In nonlinear periodic fields, waves from the center of the source and its periphery are focused almost to the geometrical focus of the source (Fig. 2a). In this case, if the pressure amplitude at the

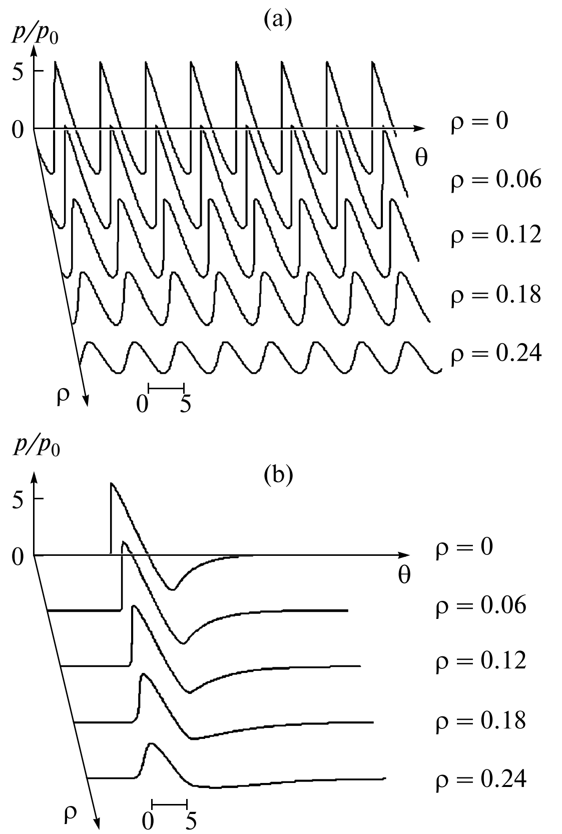

Fig. 6. Waveforms at different transverse distances ρ to the source axis at $\sigma = 0.8$ in the cases of (a) periodic and (b) pulsed fields.

source grows, saturation occurs in the central region of a beam, but from the source periphery, where saturation is not attained yet, waves arrive. This is precisely the reason that the saturation curve for a periodic field in the range of the studied parameters does not stabilize at a constant level and the value of the peak pressure in the focus continues to grow slowly (Fig. 5a).

In contrast to periodic fields, the phenomenon of nonlinear refraction is very significant in pulsed fields. Waves from the central part of the source are focused behind the geometrical focus due to nonlinear refraction, while weaker waves from the beam periphery still come approximately to the geometrical focus of the source. Thus, in distinction to a periodic field, in a pulsed field the waves from the central part of the source and from the source periphery are focused at different points and, therefore, they do not amplify each other. This leads to the fact that, in pulsed fields, a saturation curve stabilizes at a constant pressure level lower in comparison with pressure in periodic beams with the same pressure amplitude at the source. Thus limitation of the peak positive pressure in pulsed fields is caused by the phenomenon of nonlinear refraction.

Figure 7 presents the dependences of the wave beam energy on the distance from the source in (a) periodic and (b) pulsed focused fields at different values of the nonlinear parameter $N = 0.2$, 0.5, 1.0, and 3.0. The energy of a pulsed field at an arbitrary distance from the source was calculated as an integral over a time window and the beam aperture in squared pressure at each point and normalized to its initial value, and the energy over a single period was calculated as the energy of a periodic wave and also normalized to its initial value,

$$\frac{E}{E_0} = \iint \frac{p^2}{p_0^2} r\,dr\,dt. \tag{12}$$

One can see from Fig. 7 that, near the source, the energy of periodic and pulsed beams remains constant. Then, starting from the distance σ corresponding to the length of shock formation in waveforms at the beam axis, the energy starts to decrease on account of nonlinear absorption in shocks. It is well known that, in a plane nonlinear wave, the energy of a pulsed signal after shock formation decreases with the distance as $1/\sigma$ and the energy of a periodic perturbation decreases faster as $1/\sigma^2$ [1]. In this case the effect of pressure saturation is observed in a plane periodic wave, while there is no saturation for a pulsed signal [5, 6]. As one can see from Fig. 7, in the case of a focused radiator, the energy of a periodic field decreases with the distance also faster than the energy of a pulsed shock-wave field. Thus nonlinear absorption at the shock front of a wave for focused periodic fields manifests itself stronger than for pulsed ones. This allows us to conclude that the major mechanism leading to saturation in focused periodic fields is nonlinear absorption. The influence of nonlinear refraction manifests itself only in a small region near the focus and is insignificant on the whole. For pulsed fields the major mechanism leading to saturation of the peak positive pressure is nonlinear refraction.

It is interesting to note that, despite the fact that the energy of a periodic beam decreases faster, the maximum attainable value of the peak positive pressure in a periodic field is larger than that in a pulsed one. This feature for manifestation of nonlinear effects indicates that pulsed beams are more suitable for transmission of wave energy to the focus with smaller losses and beams of periodic waves, for attaining high pressure values in the focal region.

CONCLUSIONS

Here we compare physical mechanisms leading to the effects of saturation in focused periodic and pulsed fields generated by sources with Gaussian spatial apodization. It has been demonstrated that saturation for the peak positive pressure in focused periodic beams occurs due to nonlinear absorption at the shock front of a wave. Pressure saturation in pulsed fields is connected with manifestation of the nonlinear refraction effect. The limiting attainable values of the peak positive pressure in periodic fields turned out to be

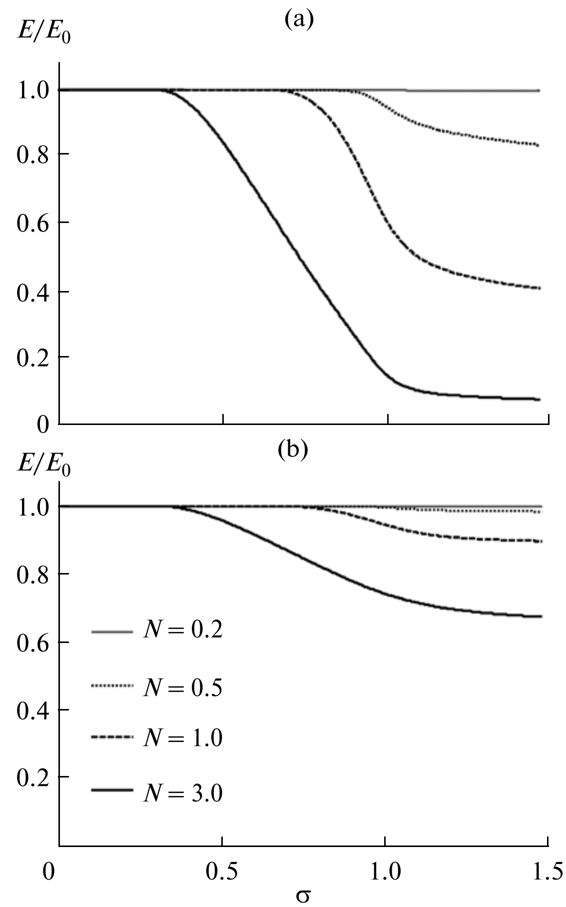

Fig. 7. Dependences of energy in a wave beam on the distance σ to the source in (a) periodic and (b) pulsed fields at different values of the nonlinear parameter $N = 0.2$, 0.5, 1.0, and 3.0.

higher than the analogous values in pulsed acoustic fields. It has been demonstrated that the saturation pressure for the peak positive pressure in pulsed and periodic fields does not depend on the initial pulse length or the frequency of the initial harmonic wave. The total energy in a beam of periodic waves decreases with the distance from the source faster than in the case of a pulsed field; however, it becomes concentrated within a significantly smaller spatial region near the focus.

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