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Controlling the Coefficient of Reflection of Sound from a Plane Piezoelectric Plate by Selecting its Electrical Load

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Abstract—An analysis is performed of the possibility of minimizing the coefficient of reflection of a plane acoustic wave from a plane piezoelectric transducer by connecting it to an electrical load with a specially selected impedance. It is shown in theory that the reflection coefficient vanishes when there are no losses in the piezoelectric element under the condition of electrical matching. The effect the impedance of the electrical load has on the coefficient of sound reflecting off a piezoelectric plate is demonstrated experimentally.

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INTRODUCITON

Piezoelectric transducers operating on the basis of the direct and inverse piezoelectric effect are traditionally used for the radiation and reception of acoustic waves in their respective modes [1]. It is known that condition of electric matching must be met for the most efficient transfer of energy from an emf source to a connected load. This requires the power transferred to the load to be as high as possible. In the transmit mode, an electric generator acts as the emf source and a piezoelectric transducer is the electrical load [2]. In the receive mode, the source of the emf is the piezoelectric transducer itself when an acoustic wave falls upon it. The efficiency of the conversion of acoustic into electrical energy then depends on the electrical impedance of the load connected to the conducting sides of the piezoelectric plate. This electrical impedance can be selected so as to obtain the maximum electrical energy transferred to the load in the form of heat. As a result, the coefficient of reflection of the incident acoustic wave from the piezoelectric plate will be the lowest possible. This way of reducing the coefficient of reflection off a piezotransducer can be useful when performing experimental studies in which it is necessary to reduce the effect of re-reflected waves that can distort measuring results [3].

The dependence of the reflection coefficient on the magnitude of a Q-factor of the electrical load of a half-wave piezoelectric element operating in the receive mode was considered in [4]. The existence of optimum matching frequencies of a piezoelectric element with a wave propagation medium for different acoustic loads was established in [5].

In this work, we analyze the possibility of controlling the coefficient of reflection off the surface of a flat piezoelectric transducer by changing its electrical load. A simple analytical expression is obtained that associates the coefficient of sound reflection from a piezoelectric plate and the impedance of a piezotransducer with an air backing at an arbitrary frequency in an approximation of no mechanical and electrical losses in the piezoceramics. For theoretical calculations and experimental studies, we used a circular plane piezoceramic transducer with a diameter of 100 mm and an air backing. Its piezoelectric and mechanical characteristics were determined experimentally in [6] by measuring its electrical impedance, which allowed us to analyze the operation of the piezotransducer with a high degree of accuracy.

THEORY

The large radius of the piezoelectric plate relative to its thickness allowed us to consider electroacoustic transformation in a one-dimensional approximation (the six-terminal network model) [2]. Considering the operation of the transducer in the transmit mode and allowing for the air backing (acoustic impedance of the load $z_1 \to 0$), we can in this case obtain the expression for the electrical impedance of the piezotransducer:

$$Z_0 = \frac{i}{\omega C_0}\left[1 - \frac{k_T^2}{kl}\,\frac{i\dfrac{z_2}{z}\sin kl + 2(1-\cos kl)}{\sin kl + i\dfrac{z_2}{z}\cos kl}\right]. \qquad (1)$$

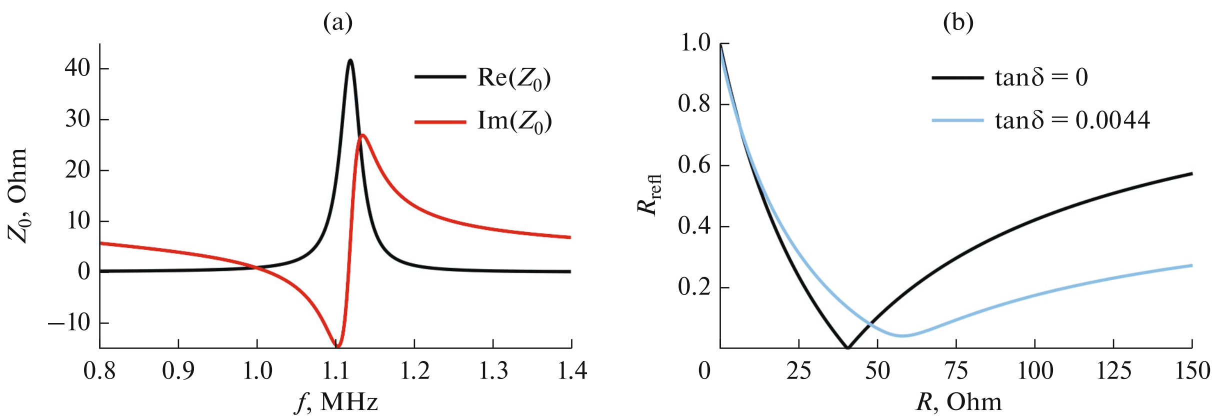

Fig. 1. (a) Real and imaginary parts of the impedance of the piezotransducer; (b) absolute value of the coefficient of reflection of an incident plane acoustic wave off the piezoplate, as a function of the active load at the frequency of antiresonance using two values of the piezoceramic's loss factor.

Here, $k = \omega/c$ is the wavenumber in piezoceramic; $\omega = 2\pi f$ is the cyclic frequency of harmonic oscillations in the system; c is the sound speed in the piezoceramic; l is the thickness of the piezoceramic plate; z and z_2 are the acoustic impedances of the plate and immersion medium (water), respectively; C_0 is the capacitance of the clamped transducer; and k_T is the electromechanical coupling coefficient. It is assumed that all processes obey harmonic law ~exp($-i\omega t$). For simplicity, we consider the losses inside the piezoelectric plate to be negligible (i.e., the tangent of the angle of the mechanical and electrical losses of the piezoceramic are assumed to be zero). Figure 1a shows plots of the frequency dependences of the real and imaginary parts of the impedance of our piezotransducer in the vicinity of its first resonance, calculated using formula (1). Values determined in [6] were used in our calculations: $c = 4500$ m s^{-1}; $l = 2.01$ mm; $k_T = 0.462$; $C_0 = 21.19$ nF; $z = 3.4 \times 10^7$ kg m^{-2} s^{-1}, $z_2 = 1.5 \times 10^6$ kg m^{-2} s^{-1}. The two characteristic frequencies at which the imaginary part of the impedance vanishes are those of resonance and antiresonance [7], where the piezotransducer operates as purely active resistance. The real part approaches its maximum value at the frequency of antiresonance, which in many cases makes it better for operating in the transmit and receive modes.

Let us consider the operation of the piezotransducer in the transmit mode, where an acoustic wave incident on the piezotransducer induces voltage on the conducting sides of the piezoplate. Theoretical analysis shows that the piezoplate acts as an electric generator with internal impedance Z_0 equal to the electrical impedance of the transducer. Calculating the coefficient of reflection of a plane ultrasound wave off a piezoplate with an air backing yields the expression

$$R_{\text{refl}} = \frac{\sin kl - i\frac{z_2}{z}\cos kl}{\sin kl + i\frac{z_2}{z}\cos kl} \times \frac{Z_{\text{el}} + \dfrac{i}{\omega C_0}\left[1 - \dfrac{k_T^2}{kl}\dfrac{-i\frac{z_2}{z}\sin kl + 2(1-\cos kl)}{\sin kl - i\frac{z_2}{z}\cos kl}\right]}{Z_{\text{el}} + \dfrac{i}{\omega C_0}\left[1 - \dfrac{k_T^2}{kl}\dfrac{i\frac{z_2}{z}\sin kl + 2(1-\cos kl)}{\sin kl + i\frac{z_2}{z}\cos kl}\right]}, \quad (2)$$

where Z_{el} is the electrical impedance of the load connected to the conducting sides of the piezoplate. In light of expression (1) for impedance Z_0, expression (2) can be converted to the more compact form

$$R_{\text{refl}} = \frac{\sin kl - i\frac{z_2}{z}\cos kl}{\sin kl + i\frac{z_2}{z}\cos kl}\,\frac{Z_{\text{el}} - Z_0^*}{Z_{\text{el}} + Z_0}, \quad (3)$$

where Z_0^* is the value of the complex conjugate to Z_0. Expression (3) clearly shows that the coefficient of reflection is equal to zero at $Z_{\text{el}} = Z_0^*$, a well-known condition of electric matching. In the limiting case of infinite electrical load $Z_{\text{el}} \to \infty$ (an open-circuit transducer), we obtain the familiar expression for the coefficient of reflection off a plane–parallel layer in [8].

Note that the first multiplier in the right-hand side of expression (3) is equal to unity in absolute magnitude, and the second multiplier depends on the value of Z_{el}. If the piezoplate is in the short-circuit ($Z_{\text{el}} \to 0$) or open-circuit ($Z_{\text{el}} \to \infty$) mode, the second multiplier is also equal to unity in absolute magnitude. In both

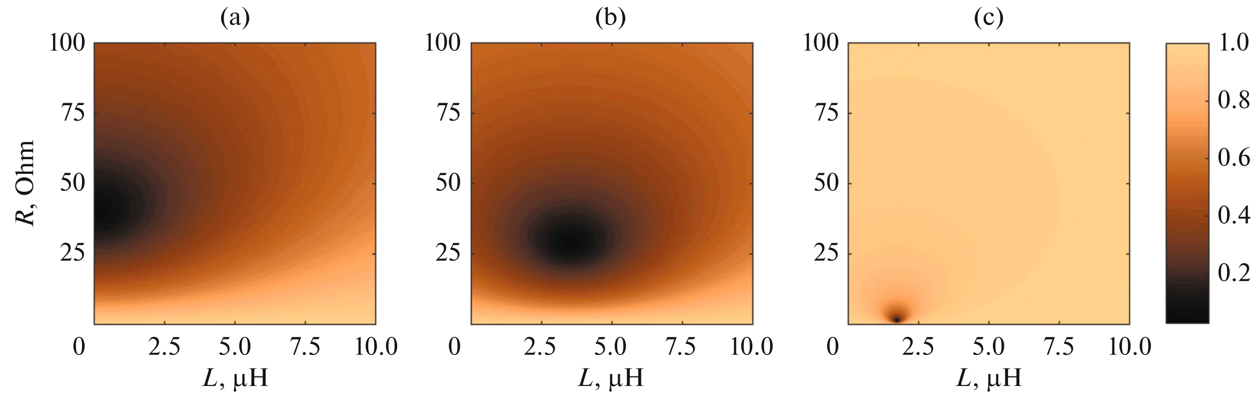

Fig. 2. Absolute value of the reflection coefficient as a function of inductance and resistance connected in parallel to the piezotransducer operating in the transmit mode at different frequencies of incident acoustic waves: (a) 1.117, (b) 1.13, and (c) 1.2 MHz.

cases, $|R_{\text{refl}}| = 1$ (i.e., there is total reflection). Reflection is not total if impedance Z_{el} of the load is different (i.e., some of the energy of the incident acoustic wave is lost in transforming into electrical energy and then into heat).

Let us write the complex electrical impedance of the piezotransducer in the form $Z_0 = X_0 + iY_0$. At the frequency of antiresonance $Y_0 = 0$ and impedance Z_0 is purely real. Since $kl \cong \pi$, formula (3) is simplified:

$$R_{\text{refl}}^a = -\frac{Z_{\text{el}} - X_0}{Z_{\text{el}} + X_0}. \quad (4)$$

We can see that in order to obtain a zero coefficient of reflection at the frequency of antiresonance, it is sufficient to select an active resistance equal to the impedance of the piezoelectric transducer, which is real at the frequency of antiresonance. Figure 1b shows the modulus of the reflection coefficient calculated using formula (2) as a function of the resistance of the electrical load connected to our piezotransducer for zero tangent of the angle of mechanical and electrical losses in the piezoceramic ($\tan\delta = 0$) and for $\tan\delta = 0.0044$. Our calculations were made at a frequency of 1.1174 MHz, which corresponds to that of the antiresonance of the piezotransducer when using an approximation of zero losses. We can see that at $\tan\delta = 0$ the coefficient of reflection falls to zero upon connecting to a resistance of 40.5 Ohm, which corresponds to the real part of impedance at the frequency of antiresonance (see Fig. 1a). The introduction of a nonzero loss factor changes the position and minimum value of the coefficient of reflection, which must be considered when selecting an electrical load to suppress the reflection of acoustic waves.

If an acoustic wave of an arbitrary frequency at which the imaginary part of the impedance does not fall to zero is incident on the transducer, inductance or capacitance (depending on the sign of Y_0) must be connected in series to active resistance in order to compensate for the imaginary part of the piezotransducer's impedance and obtain a zero reflection coefficient (see formula (3)).

Let us consider a case where the frequency is slightly higher than that of antiresonance. Then $Y_0 > 0$, and to minimize the coefficient of reflection we must use inductance L in combination of active resistance R: $Z_{\text{el}} = R - i\omega L$. Figure 2 shows the dependence of the modulus of the coefficient of reflection on resistance and inductance at the frequency of antiresonance (on the left-hand side) and at two frequencies exceeding the frequency of antiresonance (the two plots on the right-hand side). If the frequency deviates from that of antiresonance, the range of resistances and inductances (capacitances) at which the coefficient of reflection becomes less than the fixed one is narrowed; for example, $R_{\text{refl}} < 0.5$ (i.e., there is a drop in stability). This complicates minimizing the reflection coefficient by performing experiments at frequencies that differ strongly from that of antiresonance.

Note that in the simplest experiment implementation, where a generator with an internal resistance of 50 Ohm is connected in parallel to a piezotransducer, the coefficient of reflection at the frequency of antiresonance is equal to 10%, according to formula (4) (i.e., the reflected wave is already suppressed quite strongly). The coefficient of reflection can be reduced to zero by modifying the impedance of the load with an additional resistor.

EXPERIMENTS

To confirm qualitatively that the coefficient of reflection can be mininimized, we performed experiments with the piezotransducer described above, operating in the modes of ultrasound pulse radiation and reception. Figure 3 shows the experimental setup

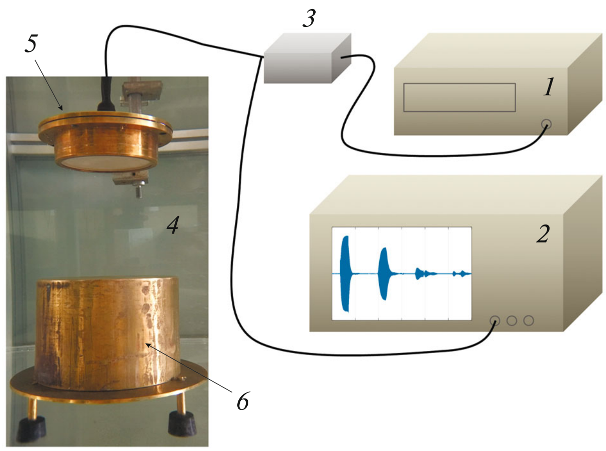

Fig. 3. Scheme of the experimental setup: (*1*) generator, (*2*) oscilloscope, (*3*) active resistance of 1 kOhm, (*4*) water tank, (*5*) piezoelectric transducer, and (*6*) brass cylinder used as a reflector.

for measuring the coefficient of reflection. A sinusoidal signal consisting of 20 or 40 periods with amplitude of 10 V at a thickness resonance frequency of 1.12 MHz was supplied from an electric generator to a piezotransducer; according to calculations, in the vicinity of this frequency the antiresonance one is located. The piezotransducer emitted an acoustic wave, which propagated to a flat reflector in the form of a brass cylinder [9] located at a distance of L = 12 cm, and was reflected from it. An oscilloscope was used to record electrical signals from the piezotransducer. The surface of the reflector was positioned parallel to the piezotransducer surface according to the maximum amplitude of the reflected signal received by the piezotransducer and observed on an oscilloscope. The internal resistance of the generator was 50 Ohm. In the piezotransducer's mode of reception, it acted as the electrical impedance loaded upon the conducting sides of the piezoplate. A resistance of 1 kOhm could also be connected in series with the generator. According to calculations, the coefficient of reflection for this resistance would be close to 1.

To determine the coefficient of reflection, we selected the first and second pulses reflected off the cylinder. Using the Fourier transform, we calculated the frequency spectra of electrical signals corresponding to the reflected waves. Electrical signals recorded by the oscilloscope repeat the time dependence of the voltage on the piezoplate conducting sides and are associated with acoustic signals (pressure in an acoustic wave) on the piezoplate by a quantity referred to as sensitivity (or the transfer function), which depends on the characteristics of the piezotransducer and the frequency [6]. The ratio of the spectra of the second and first reflected signals on the oscilloscope are thus equal to the ratio of the spectra of the second and first reflected acoustic waves arriving at the piezotransducer. In the considered approximation of a one-dimensional piezotransducer, spectrum $S_{p2}(f)$ of the second reflected wave is therefore related to spectrum $S_{p1}(f)$ of the first reflected wave:

$$S_{p2}(f) = K_{brass} R_{refl}(f) S_{p1}(f), \quad (5)$$

where f is frequency; $K_{brass} = R_{brass} R_{diffr}$; R_{brass} is the coefficient of reflection off brass ($R_{brass} = 0.93$ when a plane wave reflects off a plane water–brass interface); and R_{diffr} is a coefficient that considers diffraction losses caused by the divergence of a wave, which can additionally reduce the measured amplitudes of the reflected signals. We shall consider R_{diffr} to be a constant in the considered narrow frequency domain. The experimentally determined quantity is then

$$\frac{S_{u2}(f)}{S_{u1}(f)} = K_{brass} R_{refl}(f). \quad (6)$$

Here, $S_{u1}(f)$ and $S_{u2}(f)$ are the spectral amplitudes of the first and second reflected signals measured with the oscilloscope.

We estimated the value of R_{diffr}, which considers the effect diffraction has on the ratio of the measured amplitudes of the reflected signals. Reflection off a flat reflector can be presented as radiation from a source located mirror-symmetrically, relative to the surface of reflection with respect to the true source. The amplitude of the first reflected wave can be estimated as the pressure averaged over the surface area of the piezotransducer at a distance of $2L$ [10]. Since only those waves that hit the piezoplate's surface are reflected off it, the amplitude of the second reflected wave can be estimated in a similar way. The ratio of the averaged amplitudes of the second and first reflected waves is an estimate of the value of R_{diffr}. This value was $R_{diffr} = 0.94$ for distance L used in our experiment (i.e., diffraction losses were 6%).

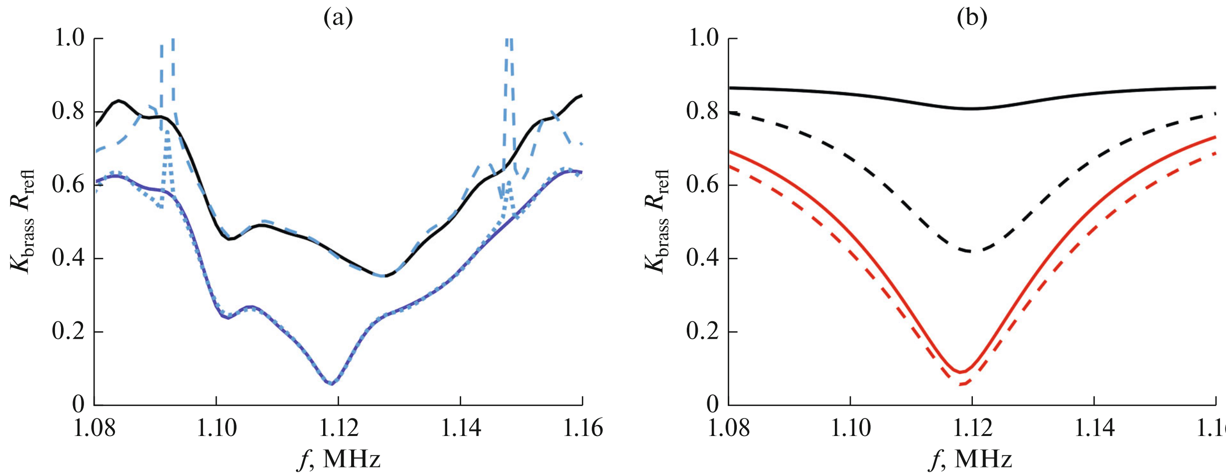

Fig. 4. (a) Ratio of the spectral amplitudes of the experimentally measured second and first reflected signals. The solid curves are results obtained for pulses containing 20 periods. The black curve is for a connected electrical load of 1050 Ohm; the blue curve, for load of 50 Ohm. The dashed and dotted curves are results obtained for pulses containing 40 periods. The dashed curve is for a load of 1050 Ohm; the dotted curve, for a load of 50 Ohm. (b) The coefficient of reflection was calculated in theory by considering losses due to reflection off brass and diffraction. The solid curves are for $\tan\delta = 0$; the dashed curves, for $\tan\delta = 0.0044$. The black curves are for electrical load of 1050 Ohm; the red curve, for load 50 Ohm.

Figure 4a shows the experimental frequency dependences of value $K_{brass}R_{refl}(f)$ for two variants of the electrical load impedance (50 and 1050 Ohm). Figures 1b and 2 show that at a load of 50 Ohm, the coefficient of reflection off the piezotransducer should be low in the vicinity of the frequency of antiresonance, as is reflected in the experimental results. A drop in spectral amplitude $S_{u2}(f)$ of the second reflected pulse is observed at a frequency of 1.118 MHz, and minimum value $K_{brass}R_{refl}(f) = 0.055$ is reached (see Fig. 4a). The observed coefficient of reflection is an order of magnitude higher at a load of 1 kOhm.

Figure 4a shows the curves obtained from experimental measurements using 20 and 40 periods of the signal from the generator. They coincide with one another, testifying to the good repeatability of the results for this configuration of the setup.

Theoretical calculations (see Fig. 4b) also showed a sharp drop in the reflection coefficient at the frequency of antiresonance at an electrical load of 50 Ohm. It was found that the value of the reflection coefficient also strongly depends on the mechanical and electrical loss factor of the piezoceramic. The solid curves in Fig. 4b are plotted for zero loss factor $\tan\delta = 0$; the dashed curves, for $\tan\delta = 0.0044$, the loss factor selected within the tabular values so that the coefficient of reflection off the piezotransducer at antiresonance would be approximately the same as the experimental values.

The experimental data thus demonstrate the possibility of affecting the coefficient of reflection by changing the electrical load of the piezotransducer, quantitatively confirming the theory and providing approximate numerical results for the coefficient of reflection. The minimum observed coefficient of reflection, obtained using experimental data and allowing for diffraction losses and the coefficient of reflection off brass, is therefore $R_{refl}(f) = 0.063$ (i.e. the selection of the electrical impedance can greatly reduce the coefficient of reflection off the piezotransducer).

CONCLUSIONS

The possibility of considerably reducing the coefficient of reflection of a plane acoustic wave from a piezoplate (to zero with no losses in the piezoelectric) by choosing the electrical load was confirmed in theory. If the frequency of antiresonance is used as the operating frequency in a loss-free system, we can completely convert the energy of an acoustic wave into electrical energy using the active resistance of the calculated value, which is determined by the real part of the electrical impedance of the piezotransducer. The possibility of affecting the coefficient of reflection by changing the electrical load of the piezotransducer was demonstrated experimentally.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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