# Modeling elastic wave propagation in kidney stones with application to shock wave lithotripsy

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A time-domain finite-difference solution to the equations of linear elasticity was used to model the propagation of lithotripsy waves in kidney stones. The model was used to determine the loading on the stone (principal stresses and strains and maximum shear stresses and strains) due to the impact of lithotripsy shock waves. The simulations show that the peak loading induced in kidney stones is generated by constructive interference from shear waves launched from the outer edge of the stone with other waves in the stone. Notably the shear wave induced loads were significantly larger than the loads generated by the classic Hopkinson or spall effect. For simulations where the diameter of the focal spot of the lithotripter was smaller than that of the stone the loading decreased by more than 50%. The constructive interference was also sensitive to shock rise time and it was found that the peak tensile stress reduced by 30% as rise time increased from 25 to 150 ns. These results demonstrate that shear waves likely play a critical role in stone comminution and that lithotripters with large focal widths and short rise times should be effective at generating high stresses inside kidney stones. © 2005 Acoustical Society of America. [DOI: 10.1121/1.2032187]

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# I. INTRODUCTION

Lithotripsy is the clinical procedure whereby extracorporeally generated shock waves are focused onto kidney stones to fragment them into small enough pieces that they can be passed naturally. Lithotripsy was first introduced in 1980 (Ref. 1) and in the United States about 70% of kidney stones are treated with shock waves.<sup>2</sup> Despite the wide acceptance of shock wave lithotripsy the mechanisms by which the shock waves comminute stones are not agreed upon. In addition it is now recognized that lithotripsy is not a benign procedure but results in tissue damage to most if not all kidneys which can lead to both acute and chronic complications.<sup>3–6</sup> Until the mechanisms of kidney stone fragmentation and kidney tissue damage are identified, improvements to lithotripsy are principally found empirically.

Identification of the mechanisms of kidney stone fragmentation requires that one first determine the state of loading on the stone and after which a fracture mechanics model needs to be applied to determine the material failure. A few models to determine the loading within kidney stones have been published. Dahake and Gracewski<sup>7,8</sup> used a finitedifference time-domain (FDTD) simulation for elastic waves but their study was motivated by intracorporeal shock wave lithotripsy and they treated the case of point sources on the surface of spherical stones. Xi and Zhong<sup>9</sup> have used ray tracing of compression and shear waves to determine locations of interactions but the ray tracing model cannot predict stress amplitudes and does not account for diffractive effects. Cleveland and Tello<sup>10</sup> described a FDTD model for calculating pressure waves in kidney stones subject to lithotripsy shock waves but the model treated the stone as a fluid and neglected the presence of shear waves. Mihradi *et al.*<sup>11</sup> employed a finite-element model (FEM) to predict the stress loadings on kidney stones in lithotripsy. The incident waveform was modeled as a half-sinusoid of purely positive pressure and simulations were carried out for pulses of different durations (0.5 to 5  $\mu$ s) in a two-dimensional Cartesian coordinate system. It was concluded that fracture occurs due to spall, that is, reflection of the positive pressure from the distal surface of the stone.

Once the state of loading is determined the criteria for failure has to be identified. Most measurements on stone strength have reported either compressive strength or fracture toughness in experiments that were carried out at low strain rates.<sup>12–14</sup> This data shows enormous variation and because the strain rate in lithotripsy is  $10^5 \text{ s}^{-1}$  it is not clear whether measurements at low strain rates provide data relevant to lithotripsy.<sup>15</sup> Also, it is likely that the fracture is a fatigue process and requires further data, such as dynamic fracture toughness.<sup>16</sup>

This paper aims to address the issue of determining the stresses and strains in kidney stones during lithotripsy. We employ a finite-difference scheme, which is widely used in the geophysics community, to model the propagation of shock waves in kidney stones. This allows us to compute the stress and strain inside stones accounting for pressure waves in the surrounding fluid and elastic waves in the solid includ-

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ing all surface interactions. Using the simulations we are able to quantify a key role of shear waves that has only been alluded to in previous work.

# **II. MODEL**

In this work kidney stones were assumed to behave as a linear, isotropic, elastic medium (the validity of this assumption is addressed in the discussion), in which case the underlying equations are Newton's second law and Hooke's law:

$$\frac{\partial v_i}{\partial t} = \frac{1}{\rho_0} \frac{\partial \tau_{ij}}{\partial x_j},\tag{1}$$

$$\frac{\partial \tau_{ij}}{\partial t} = \lambda \,\delta_{ij} \frac{\partial v_k}{\partial x_k} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \tag{2}$$

The equations are written in index notation where  $v_i$  represent the velocity at each location,  $\tau_{ij}$  the stress tensor,  $\delta_{ij}$  the Kronecker delta function, a repeated index implies summation,  $\rho_0$  is the material density, and  $\lambda$  and  $\mu$  are the Lame coefficients of the material.

We further assumed that the problem was axisymmetric and cast the equations in cylindrical coordinates  $(r, \theta, z)$ . The z axis was the axis of propagation of the shock wave and the stones were modeled as volumes of rotation around the z axis (in this paper either spheres or cylinders). For cylindrical coordinates  $v_{\theta}=0$  and all derivatives with respect to  $\theta$  vanish (see, for example, Ref. 17, pp. 26–27; Ref. 18, p. 77; or Ref. 19, Chap. 12) in which case Eq. (1) becomes

$$\frac{\partial v_r}{\partial t} = \frac{1}{\rho_0} \left( \frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} \right),$$

$$\frac{\partial v_z}{\partial t} = \frac{1}{\rho_0} \left( \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} \right),$$
(3)

and the nonzero stresses in Eq. (2) are given by

$$\frac{\partial \tau_{rr}}{\partial t} = \lambda \, \nabla \cdot \mathbf{v} + 2\mu \frac{\partial v_r}{\partial r}, \quad \frac{\partial \tau_{zz}}{\partial t} = \lambda \, \nabla \cdot \mathbf{v} + 2\mu \frac{\partial v_z}{\partial z},$$

$$\frac{\partial \tau_{\theta\theta}}{\partial t} = \lambda \, \nabla \cdot \mathbf{v} + 2\mu \frac{v_r}{r}, \quad \frac{\partial \tau_{rz}}{\partial t} = \mu \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right),$$
(4)

where the dilatation rate is

$$\nabla \cdot \mathbf{v} = \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z}.$$
 (5)

Equations (3)–(5) were solved using a finite-difference grid that was staggered in both time and space, referred to as the Vireux scheme in geophysics<sup>20–22</sup> or the Yee cell in electromagnetics (Ref. 23 and Ref. 24, pp. 107–144). In this formulation it is assumed that the velocities are known at time t– $\Delta t/2$  and the stresses known at time t. To march forward one step first the velocities are advanced from  $t-\Delta t/2$  to t+ $\Delta t/2$  using Eqs. (3) and in the second the stresses are advanced from t to  $t+\Delta t$  using Eqs. (4) and (5). Due to the nature of the staggered grids the  $v_r/r$  term is never evaluated at r=0 and therefore does not need special attention within the algorithm. The material properties were allowed to vary arbitrarily in space and were assumed to be defined at the same spatial location as the normal stresses. Linear interpolation was used to determine material properties at staggered grid points.

The edges of the numerical grid produce reflections that can be avoided by either solving on a large enough domain that the reflections are delayed enough in time that they can be gated out of the analysis or by use of boundary layers that absorb incident waves in particular the perfectly matched layer (PML).<sup>25,26</sup> In these 2D simulations (where computer memory limitations are not a significant issue) the grid was chosen to be very large (at least 8 mm away from any interface) to ensure that reflections did not enter the domain of interest (the stone) during the time of interest. For these simulations the boundary condition applied along each boundary was that the shear stress and normal component of the velocity were zero which ensured that along the r=0 axis the axis-symmetric assumption in cylindrical coordinates was satisfied.

From a material failure point of view, the principal stresses and strains and maximum shear stress and strain are important indicators of fracture. Two of the principal stresses lie in the r-z plane and are given by

$$\sigma_{1,2} = (\tau_{rr} + \tau_{zz})/2 \pm \sqrt{\tau_{rz}^2 + (\tau_{rr} - \tau_{zz})^2/4}$$
(6)

with the angle of  $\sigma_1$  with respect to the *r* axis given by

$$\tan 2\theta_{p1} = 2\tau_{rz} / (\tau_{rr} - \tau_{zz}).$$
<sup>(7)</sup>

Due to the absence of the other two shear stresses the third principle stress is  $\sigma_3 = \tau_{\theta\theta}$ . We define the maximum tensile stress as  $\sigma_T = \max(\sigma_1, \sigma_3)$  and the peak compressive stress  $\sigma_C = -\min(\sigma_2, \sigma_3)$  (large compression is associated with a large negative number). The maximum shear stress is given by

$$\tau_{\max} = (\max(\sigma_i) - \min(\sigma_i))/2.$$
(8)

The principal strains  $u_i$  are related to the corresponding principal stresses  $\sigma_i$  by  $u_i = (1/2\mu)\sigma_i + A$ , where  $A = p/(2\mu + 4\mu^2/3\lambda)$  is a scalar value proportional to the effective pressure in the stone  $p = -(\tau_{zz} + \tau_{rr} + \tau_{\theta\theta})/3$ .<sup>27</sup> This means that the maximum and minimum strains are oriented in the same directions as the corresponding maximum and minimum stresses. Note that maximum strain associated with shear waves does not depend on p, that is, it is proportional to maximum shear stress.

To aid interpretation of the results we will employ the fact that the divergence of the particle velocity [Eq. (5)] can be used to identify the disturbances that travel at the longitudinal sound speed. Similarly the curl of the particle velocity  $\nabla \times \mathbf{v}$  can be used to identify disturbances that travel at the shear (transverse) wave speed. In our case only the  $\theta$  component  $\omega$  is nonzero:

$$\omega = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}.$$
(9)

Tracking the divergence and curl allows the propagation of different types of waves within kidney stones to be followed.

For one set of simulations we were interested in solving the elastic equations in Cartesian coordinates. We used Eqs.

TABLE I. Physical properties of media used in the simulations. The properties  $\rho$ ,  $\lambda$ , and  $\mu$  were the three independent properties that were input to the code.

	$\rho$ (kg/m <sup>3</sup> )	$c_L$ (m/s)	$c_S (\text{m/s})$	$\lambda  (\text{GPa})$	$\mu$ (GPa)	υ
Water	1000	1500		2.25	0	0.500
PSM-9	1150	2493	1108	4.32	1.41	0.377
Solid E30	1700	3000	1500	7.65	3.83	0.333
Fluid F30	1700	3000		15.30	0	0.500

(3)–(5) with the substitution  $(r, \theta, z) \rightarrow (x, y, z)$  and all terms with 1/r drop from the equations. We assumed a state of plane strain in which case the stress in the out-of-plane axis (y) is given by  $\tau_{yy}=-\tau_{xx}+\tau_{zz}/v$  where  $v=\lambda/[2(\lambda+\mu)]$  is Poisson's ratio. All the other field variables can be calculated using the same formulas given above.

# A. Material properties

The model requires three material properties for each medium—the density and the two Lame coefficients. The most common measurements made on natural and artificial kidney stones are the density, longitudinal sound speed, and shear wave speed which are related to the Lame coefficients through

$$c_L = \sqrt{\frac{\lambda + 2\mu}{\rho_0}}, \quad c_S = \sqrt{\frac{\mu}{\rho_0}}.$$
 (10)

In Table I we show the material properties used in the simulations here.

In the simulations presented here we considered stones surrounded by water, which is a reasonable approximation of both *in vitro* and *in vivo* environments in which stones are destroyed. The material properties over the entire grid were therefore first initialized to that of water. In this work we considered stones of either cylindrical or spherical shape with homogeneous internal structure. Once the location and geometry of the stone were specified the indices on the grid that fell within the stone volume were identified and then  $\rho$ ,  $\lambda$ , and  $\mu$  for those indices set to the properties of the stone.

#### **B.** Source condition

The source condition was taken to be a classic lithotripsy pulse<sup>28</sup> modified with a hyperbolic tangent function to provide a smooth shock front:

$$p_s(t) = 0.5(1 + \tanh(t/t_{RT})) \exp(-t/t_L) \cos(2\pi f_L t + \pi/3),$$
(11)

where  $t_L=1.1 \ \mu s$  and  $f_L=83.3 \text{ kHz}$  control the pulse shape and  $t_{RT}$  controls the rise time which was varied from 50 to 200 ns. The source was input in one of two ways. (1) The source was modeled as a boundary condition in a plane placed in the fluid at a distance of 5 mm from the stone. The source pressure was coupled into the simulations by setting the normal stresses in the source plane to  $\tau_{rr}=\tau_{zz}=\tau_{\theta\theta}=-p_S$ , where the negative sign accounts for the fact that in solid mechanics a compressed material has negative stress where as in fluid mechanics a compressed material is under positive pressure. The shear stress in the fluid was zero. (2) The source was modeled as an initial condition with the pressure distributed in space, obtained by replacing t in Eq. (11) by  $-z/c_0$ , and the particle velocity was determined based on the plane wave relationship  $v=p/(\rho c_0)$ . The results were insensitive to the manner in which the source was included.

Most of the simulations considered the incident pulse as a homogeneous plane wave (that is, the amplitude of the pulse did not vary with r). In addition a finite-sized focal zone was modeled by applying a Gaussian shading as a function of radial distance to the amplitude of the pulse:

$$p_s(r,z,t) = \exp(-(r/r_G)^2)p_s(0,z,t),$$
(12)

where the effective diameter (half pressure) of the focal spot is  $1.66r_G$ . No phase correction was applied as in the focal region of a lithotripter the wavefronts are close to planar.

# **III. RESULTS**

In what follows the code is first used to identify the role that shear waves play in determining the peak stresses and strains in a finite length cylindrical stone and a spherical stone. The code is then compared against published experimental data. Finally, the code is used to determine how the peak stress inside a stone is affected by (1) the size of the shock wave focal spot, (2) the size of the stone, and (3) the shock wave rise time.

A cylindrical stone was chosen to match artificial stones reported in the literature<sup>29</sup> (diameter 6.5 mm, length 7.5 mm, and material properties identified by E30 in Table I). Figure 1 includes snapshots of the divergence and the curl, which show disturbances propagating at the longitudinal wave and shear wave speed, respectively. Also shown are the maximum tensile stress and the maximum shear stresses. The top row (2.8  $\mu$ s) corresponds to when the leading compressive phase of the incident wave has almost reached the distal surface of the stone. It is seen that there are almost no longitudinal perturbations after the shock wave passage (see divergence images). However, at the edges of the stone a shear wave is produced (see curl images) and in addition an inverted diffracted compression wave is produced-analogous to the edge wave from finite amplitude aperture. Because of the higher sound speed in the stone the compression wave inside the stone runs ahead of the wave in the fluid and so to ensure the boundary conditions with the fluid a shear wave is generated in the stone at the boundary. Because the speed of the longitudinal wave is higher than that of the shear wave the effective source of shear waves at the stone boundary is "supersonic" and results in a conical wavefront (similar to a sonic boom). The longitudinal wave in the stone also launches an acoustic wave in the fluid which also has a conical wavefront. The incident wave propagating in the fluid outside of the stone induces a stress in the stone that also produces a shear wave in the solid but because the speed in the fluid is the same as the shear wave speed the wavefront is curved. Finally the passage of the pressure wave in the fluid over the outer surface of the stone generates a surface wave in the stone at the stone-water interface-this is a coupling



FIG. 1. (Color) Snapshots of the divergence, curl, max tensile stress, and maximum shear stress inside a cylindrical stone at 2.6, 3.2, 3.6, 3.8, and 4.8  $\mu$ s after the SW is incident on the stone. The shock wave is incident from beneath the stone.

of shear and longitudinal waves and can be seen in both the divergence and curl images. This wave is restricted to a small region (<0.5 mm) near the interface.

When the wavefronts reflect at the distal surface  $(3.2 \ \mu s)$  a rich interaction between compressional and shear waves occurs. The compression wavefront is not planar when it is incident on the distal surface, as the presence of the fluid has "slowed down" and attenuated the outer por-

tions of the wavefront. Therefore it reflects primarily as a compression wave but also partially as a shear wave. The next wave incident on the rear surface is the conical shear wave which is close to a plane wavefront and so when it interacts with the distal surface it does reflect as shear wave. However, the second shear wavefront does have curvature and so although it reflects primarily as a shear wave it also mode converts into a compression wave. The surface wave



FIG. 2. (Color) Snap shots of the divergence, curl, max tensile stress, and maximum shear stress inside a spherical stone at 2.8, 3.8, 5.2, and 6.2 µs.

reflects too; it generates a diffracted shear wave at the corner and also launches a shear wave along the distal surface.

We now consider what happens to the maximum principal stresses (tensile stress) and maximum shear stress in the material. The numerical results indicate that the maximum tensile stress does not occur due to the interaction of the reflected compressive wave with the incoming tensile phase  $(3.2 \ \mu s)$ . Rather the peak tensile stress occurs due to an interaction of the reflected longitudinal wave with a shear wave generated from the outer surface of the stone (3.8  $\mu$ s). This shear wave is generated due to the interaction of the shock wave in the water passing the outside of the stone. This interaction also results in the highest shear stress. We also find that a large tensile stress is developed in a region localized near the lateral surfaces of the stone (3.6  $\mu$ s) due to an interaction of a shear wave reflected from the rear surface of the stone and the wave traveling in the fluid outside the stone.

Figure 2 shows analogous snapshots for a spherical stone made of the same material. At 2.8  $\mu$ s the longitudinal wave has passed the stone with a part of it having reflected off the distal surface and focused about 1.5 mm from the rear surface. At 3.8  $\mu$ s the shear wave generated by the internal compression wave **traveling inside** the stone (where it inter-

acts with the surface) focuses to a point about 2 mm from the distal surface. At 5.2  $\mu$ s the shear wave generated by the wave **traveling outside** the stone arrives at the distal end and results in the largest tensile stress. At 6.2  $\mu$ s the surface waves reach the distal point (aphelion) of the stone and generates both a large tensile stress and shear stress at the distal surface.

Figure 3 shows snapshots of the peak tensile strain in the cylindrical and spherical stones at the time 3.8  $\mu$ s—this time was when the highest strain occurred in the simulations. By comparing these images to the snapshots in Figs. 1 and 2 at the same time it can be seen that the spatial distribution of the peak tensile strain is almost identical to that of the peak tensile stress. That is, the same phenomenon that produces the highest tensile stress also resulted in the highest tensile strain of the peak compressive strain and the peak shear strain were also equivalent to the peak compressive stress and peak shear stress, respectively.

Therefore, for both the spherical and cylindrical geometry, it is the arrival of shear waves generated by the passage of pressure waves in the fluid outside of the stone that results in the highest stresses and strains in the stone. In what fol-



FIG. 3. Snapshot of the peak tensile strain (%) at 3.8  $\mu$ s for the cylindrical stone (upper) and spherical stone (lower). The distribution of the tensile strain corresponds closely to the equivalent tensile stress shown in Figs. 1 and 2, respectively.

lows data are only presented for the principal stresses but the results equally well apply to the principal strains.

The results from the simulations were compared to measurements of the stress induced by a lithotripsy shock wave in a photoelastic material (PSM-9).<sup>9</sup> In the photoelastic images the number of fringes is proportional to the difference in the principal stresses in the material (which is equivalent to the maximum shear stress). In the water the presence of the pressure waves leads to a shadowgraph effect. We simulated the experimental setup solving the stress equations on a twodimensional Cartesian grid assuming plain strain as discussed in Sec. II using the nominal material properties reported in Ref. 9 (reproduced in Table I). In Fig. 4 we compare the images of the measured fringes with the prediction of the peak shear stress from the model described here (rotated and scaled to match the published photoelastic images) and it can be seen that the features depicted in the photoelastic measurements are captured by the elastic wave model. For example, at 10  $\mu$ s (experiment time 186  $\mu$ s) the head wave in the fluid, the high stresses near the equator of the stone, and the circular distribution of stress near the distal wall of the stone are in good agreement. The simulations do appear to properly capture the stress field of this experiment.

The observation that the interaction of the shock wave with the outer surface of the stone leads to high tensile stress implies that for lithotripters with small focal zones (less than the diameter of a stone) there will not be strong generation of shear waves and so the constructive interference of large stresses may not occur. We considered the effect of the diameter of the focal zone by introducing Gaussian shading to the incident shock profile and keeping the pressure amplitude on axis (r=0) the same. In Fig. 5 we show the dependence of the peak principal stresses as a function of the diameter of the shock wave. The peak stress was determined by detecting the maximum stress for each point in the grid over the course of the simulation and then taking the largest of the maximum stresses within the volume of the stone.

All of the peak stresses decreased dramatically as the focal zone diameter decreased. For the spherical stone, as the focal diameter decreased from 11 to 4 mm all of the peak stresses were reduced by at least 50%. For the cylindrical stone the peak stresses had halved once the focal diameter was 5 mm. The result was consistent for stones of properties covering the range reported for natural stones:  $c_L$  between 2500 and 4000 m/s and v between 0.33 and 0.37 (data not shown).

We found that the generation of high stresses in the stone was not strongly dependent on the stone size. Figure 6 shows the peak stresses as a function of the diameter of a spherical stone for the case of an incident plane wave. There was little change in the peak stress as a function of diameter. We note that this is in disagreement with the results of the pressure simulations (where the stone modeled as an effective liquid) which predicted that the peak negative pressure (equivalent to tensile stress) in the stone should decrease dramatically for stones less than 4 mm in diameter.<sup>30</sup> The explanation for this discrepancy is that the pressure code does not capture shear waves in the stone, which play a dominant role in determining the peak stress. In Fig. 7 the spatial distribution of the peak tensile stress is shown for spherical stone treated as either a fluid or an elastic solid. There are both quantitative and qualitative differences in the spatial distribution and the amplitude of the stress in the stone. This is further evidence of the importance of shear waves in determining the state of stress in the stone.

We finally considered the effect of shock rise time on the stresses induced in the stone. The rise time was controlled using the  $tanh(t/t_{RT})$  term in Eq. (11) and the waveform was scaled to ensure the same peak pressure. The calculated peak stresses in the cylindrical stone are shown in Fig. 8. We see that as shock rise time increased all of the peak stresses (tensile, compressive, and shear) in the stone decreased. This occurs due to two effects: (1) The leading compressional wave in the stone suffers more from diffraction due to the longer rise time and so the peak pressure of the tensile wave reflected off the back of the stone ("spall wave") is about 31 MPa for a 50-ns rise time but only 12.9 MPa for a 200-ns rise time. (2) The spatial extent of the shear waves generated by the passage of the wave outside the stone is



FIG. 4. Comparison of the photoelastic images from PSM-9 (Fig. 9 from Ref. 9) with predictions of the maximum shear stress in the stone and the pressure in the surrounding fluid. Experimental data are shown in the middle two rows. The top row shows snapshots from the simulation corresponding to the upper row of experimental data. The bottom row shows snapshots from the simulation corresponding to the lower row of experimental data.

related to the spatial extent of the shock front. For a shock front with a 50-ns rise time the resulting shear wave had a length of 0.36 mm and for a 200-ns rise time the length was 0.66 mm. These lengths were determined from measurements of the shear stress predicted by the FDTD simulations. In the latter case the shear wave was so extended in space

that near the surface it was difficult to isolate it from the surface wave. The shorter length shear wave (associated with short rise time) is focused more efficiently along the axis of the stone and so results in stronger focusing and higher stresses than the longer length shear wave (associated with longer rise time). Therefore a shorter rise time shock wave



FIG. 5. Peak stresses (tensile, compression, and shear) as a function of diameter of the focal zone. Top: spherical stone 6.5 mm in diameter. Bottom: cylindrical stone 6.5 mm in diameter and 7.5 mm long.

yields larger stresses because the shear waves generated at the lateral side of the stone focus more strongly and the "spall wave" is higher in amplitude.

# IV. DISCUSSION AND CONCLUSIONS

We used a finite-difference time-domain solution of the linear elasticity equations to determine the evolution of the state of stress and strain inside kidney stones subject to a lithotripter shock wave. The model accounted for all wave interactions at fluid-elastic boundaries and the interaction of the various types of waves that the solid structure can support. From the model it was possible to determine the peak stresses and strains (compressive, tensile, and shear) in the stone, which are often important indicators of material failure.

The key conclusion from the simulations is that the peak stresses and strains in a kidney stone are critically dependent on the presence of shear waves. In particular for the studies here we found that the passage of the shock wave in the fluid



FIG. 6. Peak stresses (tension, compression, and shear) inside a spherical stone as a function of stone diameter. The peak stresses are relatively insensitive to stone diameter.

surrounding the stone generated shear waves or surface waves at the "equatorial" surface of the stone which propagated into the stone and interfered constructively with other waves to generate the peak stresses. This conflicts with the conclusions of Mihradi *et al.*<sup>11</sup> that the classical spall or Hopkinson effect is responsible for the peak tensile stresses in the stone. The probable explanation for this is that the waveform they used did not have a short shock front and as we show in Fig. 8 the peak tensile stress is reduced dramatically as the rise time increases.

Therefore, to obtain efficient generation of high stress inside a kidney stone it is desirable to have a high-amplitude pressure wave passing on the outside of the stone. This conclusion is identical to that of squeezing that has been postulated by Eisenmenger,<sup>31</sup> however the rationale for our conclusion is different. In squeezing it is postulated that the pressure wave in the surrounding fluid acts as a compressive hoop stress which, based on a static model for stress, generates splitting of the material. In our model the shock wave in the fluid launches shear waves and surface waves which constructively interfere to generate high stresses. The role of shear waves generated at the outer surface of the stone has also been identified using ray tracing,<sup>9</sup> but, although ray tracing can identify possible wave-wave interactions, it cannot predict the amplitude of the stresses induced by the waves. In this work we were able to show quantitatively that the role of the shear wave generated by the external shock wave is dramatic. In particular, as the diameter of the focal spot of the shock wave reduced from 11 to 4 mm, the peak stresses reduced by at least a factor of 2. This result was robust to stones of different size and material properties. We note that the stone geometries simulated here had cylindrical symmetry which should maximize the constructive interference on axis. This effect may be less pronounced for natural stones with more complex geometries but we still anticipate that shear wave effects will play an important role.

In addition, we found that peak stresses increased as the rise time of the shock wave decreased. As rise time increased from 50 to 200 ns the peak stresses decreased by approximately 30%. This was attributed to two processes: (1) the spatial extent of the shear wave increased and so the shear wave could not focus as efficiently on axis and (2) the compressive wave in the stone was affected by diffraction and the amplitude of the tensile wave reflected from the distal surface was less. We speculate that this finding may explain clinical reports that stones trapped in the ureter are difficult to break.<sup>32</sup> When the shock wave passes through the tissue the absorption of the tissue will lengthen the shock rise time to around 100 ns.<sup>33</sup> For stones in the collecting system of the kidney there is a volume of urine surrounding the stone and as the shock wave propagates through the urine it can "heal," recover a very short rise time, due to nonlinear distortion. The "healing" distance can be calculated as the shock formation distance<sup>34</sup> and for a lithotripter shock wave in water is less than 5 mm. Therefore, even a small amount of fluid around the stone will allow the shock rise time to recover. For stones in the ureter there is little or no urine surrounding the stone and therefore the shockwave cannot heal and will impact the stone with a relatively long rise time. We ac-





knowledge that the difficulty in breaking stones in the ureter can also be explained by invoking cavitation.

We note that there are limitations of the model employed here including the use of cylindrical symmetry, homogenous stones, and linear elasticity. The use of cylindrical symmetry can be relaxed at the cost of higher computational burden and the use of an absorbing boundary condition, such as the Beringer PML, will likely be required. Extension to inhomogeneous stones is relatively straightforward as it simply requires altering the material properties of the stone but this was beyond the scope of this study. The linear approximation is justified within the stone as the propagation distances are quite short. Nonlinear distortion is cumulative and is important over the range of propagation through the tissue to the stones (greater than 100 mm) but less important in the stone (propagation distances less than 10 mm). Further, incorporation of nonlinear elasticity is confounded by a paucity of data for the higher order elastic constants of kidney stones. Most information on the properties of kidney stones results from ultrasonic measurements which invoke linear elasticity theory or axial loading which cannot determine all the higher order constants. However, it is not unreasonable to assume that most kidney stones are brittle in nature and therefore can be reasonably approximated using linear elasticity up to the point of failure. Also alternative failure criteria could be em-

![](_page_8_Figure_4.jpeg)

FIG. 8. Variation of peak stress inside a cylindrical stone as a function of the shock rise time. All the peak stresses decreased as the rise time increased. The second set of curves show calculations carried out on a 10- $\mu$ m grid which show approximately a 10% increase in peak stresses for a rise time of 50 ns.

ployed, such as von Mises and Tresca conditions or fracture toughness, rather than the maximum stresses and strains calculated in this work. Finally, we note that viscoeleasticity can be incorporated into the model—which could be of particular importance to the binder phase of kidney stones.<sup>15</sup>

In conclusion the results indicate that lithotripters with large focal widths and short rise times will result in high peak stresses inside the stone. If direct stress is responsible for stone comminution, then a large-focal-zone short-risetime lithotripter should result in the best comminution of the stone. We note that the gold standard in clinical lithotripsy is the Dornier HM3 which has these characteristics. However, the trend among current clinical lithotripters is towards small focal zone lithotripters (diameters less than 5 mm) with high pressure amplitudes but not necessarily short rise times. The results from our simulations indicate that these characteristics may not be advantageous.

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