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Surface Instability and Thermal and Cavitational Phenomena as Mechanisms of Drop Explosions in Acoustic Fountains

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Abstract—A theoretical analysis is performed of the surface instability and acoustic and temperature fields in an acoustically excited drop of liquid. Such drops form in an acoustic fountain created under the action of an ultrasonic beam focused on the free boundary of the liquid. A model of a solitary spherical drop is used to describe the considered phenomena. This model is a resonator with an acoustically soft boundary, inside which a spherically symmetrical nonlinear acoustic field evolves. Mechanisms of drop explosions are proposed.

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INTRODUCTION

Means of non-invasive diagnostics and surgery with the use of ultrasound are now increasingly in demand. A new generation of medical devices, including ones that use high-intensity focused ultrasound (HIFU) to destroy benign and malignant tumors [1], deliver drugs using contrast agents [2], and stop internal bleeding [3] are actively being developed and improved.

Most HIFU procedures are based on thermal effects initiated by the absorption of ultrasound in biological tissue. However, there are also mechanical effects that are initiated by the dynamics of HIFU-induced bubbles used to destroy tissue. Sources with a sequence of pulses rather than continuous signals are normally used in devices for mechanical destruction. This is necessary to minimize thermal effects and the predominance of mechanical effects from the activity of bubbles. This means of tissue destruction is referred to as histotripsy. Developing new approaches to histotripsy so that the mechanical destruction of tissue is reliable and reproducible is definitely of considerable interest.

Studies have shown that the formation of acoustic microfountains is observed in histotripsy boiling, along with other instances of the physical destruction of tissue. Acoustic microfountains are broken into drops during their evolution. Atomization subsequently develops inside these drops [4]. Studying the general principles of acoustic spouting is thus relevant and will help elaborate the features of this particular phenomenon. A fairly simple way of observing an acoustic fountain is to generate it at a water–air interface under the effect of focused ultrasound in the megahertz range of frequencies.

High-speed filming of acoustic fountains in studies by different independent research groups [5, 6] has shown that a jet disintegrating into a chain of drops with identical sizes (approximately equal to the length of the acting ultrasonic wave) erupts from a liquid shortly after the ultrasound source is turned on. After some time, drops begin to lose their stability and explode, usually starting from the top drop. This results in dispersion (atomization) of the liquid. It has also been found that a dark dot appears in the center of a transparent drop just before an explosion. This could indicate rupturing of the liquid (i.e., cavitation). In addition, the drops themselves become turbid before mist caused by the spraying of the liquid begins to form around the drops. This is explained by the emergence of a cloud of microbubbles in the drop volume, or by the curvature of the drop surface on the micron scale. We have developed a theoretical model of this problem whose analytical solutions exist only in the context of simplified models. Numerical modeling is used to describe the problem more fully.

TEMPERATURE IN THE CENTER OF A DROP OF AN ACOUSTIC FOUNTAIN

A theoretical model of the dynamics inside a drop of an acoustic fountain is presented in [7]. This model is based on considering the acoustic field in the form of a standing wave whose structure varies slightly at times on the order of its period. Rapid changes are distinguished from slow ones using a basis of weakly

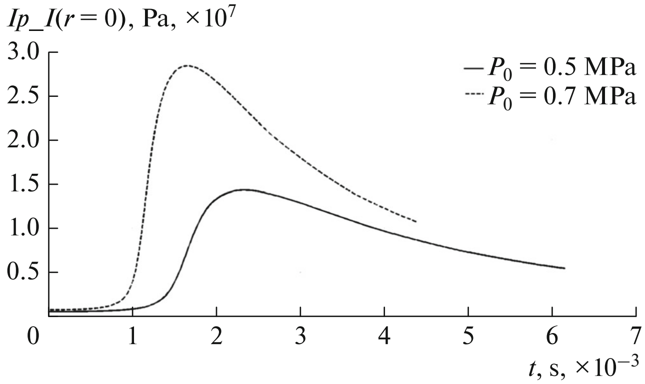

Fig. 1. Peak pressure in the center of a drop.

interacting standing waves with different frequencies in order to analyze the patterns of the nonlinear wave process in explicit form. It is found that a standing wave takes the form of an alternately converging and diverging pulse with a peak in the process of nonlinear evolution. The peak pressure in this pulse near the resonator's center can far exceed the initial amplitude of the wave. Extremely high concentrations of energy can be achieved near the resonator's center at a certain time interval despite the total loss of energy.

Viscous absorption in a drop heats the liquid, due to the transformation of acoustic energy into heat. Since the energy of higher harmonics is localized near the center of the drop, the release of heat is most efficient at the very center of the drop. The energy dissipated in a unit of volume per unit of time (on average over a period) is equal to the sum of the contributions from each harmonic:

$$Q_{\text{heat}} = \frac{b}{4\rho_0^2 c_0^2} \times \sum_{n=1}^{\infty} k_n^2 |P_n|^2 \left\{ \frac{1}{(k_n r)^2} - \frac{\sin(2k_n r)}{(k_n r)^3} + \frac{\sin^2(k_n r)}{(k_n r)^4} \right\}. \tag{1}$$

Here, b is a coefficient of dissipation; ρ_0 is the equilibrium density of a medium; c_0 is the speed of sound; r is the distance from the drop's center; k_n is the wave number of the decomposition of the acoustic field potential over the basis of functions $\sin(k_n r)/(k_n r)$; and P_n is the amplitude of the acoustic pressure harmonics in the drop.

Let T be the temperature increment of the relative initial equilibrium level, $\chi = \kappa/\rho_0 C_p$ be the thermal diffusivity, κ be the coefficient of thermal conductivity, and C_p be the specific heat capacity of the liquid. The heat balance equation has the form

$$\frac{\partial T}{\partial t} - \chi \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{Q_{\text{heat}}(r,t)}{\rho_0 C_p}. \tag{2}$$

The region of heat source localization is extremely small, so the effect of heat diffusion can be notable; the contribution from thermal conductivity must therefore be taken into account. This is easy to do by expanding the temperature increment in spatial harmonics:

$$T(r,t) = \sum_{n=1}^{\infty} \theta_n(t) \frac{\sin k_n r}{k_n r}. \tag{3}$$

Here, $\theta_n(t)$ are the amplitudes of the corresponding harmonics. Calculations of the temperature based on this approach showed that the predicted heating in the center of the drop was no higher than fractions of a degree under conditions characteristic of the drops of an acoustic fountain.

CONDITIONS FOR CAVITATION

At the initial stage, the oscillations of the drop are harmonic, and the pressure in the center is relatively low. Higher harmonics arise over time due to acoustic nonlinearity, and the profile becomes non-sinusoidal. Negative pressure levels of tens and hundreds of megapascals are reached in the center of the drops upon the formation of acoustic fountain created in practice. This is close to or exceeds the levels of liquid tensile strength; their maximum for water is 30 MPa (see Fig. 1).

We calculated how bubbles with different initial radii behave in the center of a drop at the moment when the temporal profile of the acoustic pressure in the center of the drop is maximally distorted. It turned out that small bubbles quickly collapse. A bubble begins to grow under the influence of pressure arising in the center of an acoustic fountain's drop with an increase in the initial radius of the bubbles, starting from a value of about 10 μm.

INSTABILITY OF A DROP'S SURFACE

The problem of the stability of the spherical shape of an acoustically excited drop that produces spherically symmetric oscillations is considered along with a description of the temperature field. Acoustic pressure on the drop's surface is assumed to be zero, ignoring the influence of the surrounding gas on the liquid. In other words, the drop is considered to be in a vacuum.

Our analysis of surface instability was based on considering the liquid dynamics in a drop's surface layer, where spherical disturbances develop. The liquid in this layer was considered to be incompressible. This was a reasonable approximation, since the pressure on the surface of a drop is zero. We cannot consider the liquid to be incompressible throughout a drop in the case of an initial spherically symmetric oscillation, since the liquid in this case must either rest everywhere (a property of solutions to the Laplace equation) or contain a source and a drain of mass (e.g.,

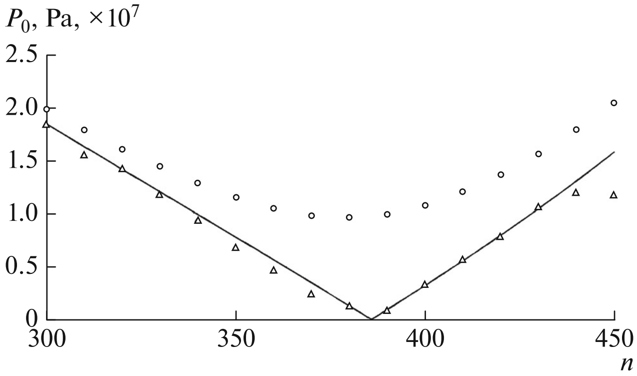

Fig. 2. First zone of instability of a drop surface in parameters P_0 (the pressure at the center of the drop at the initial moment): n is a harmonic number; the black line is for theoretical calculations without viscosity; Δ represents numerical calculations without viscosity; ○ denotes a numerical calculations with viscosity.

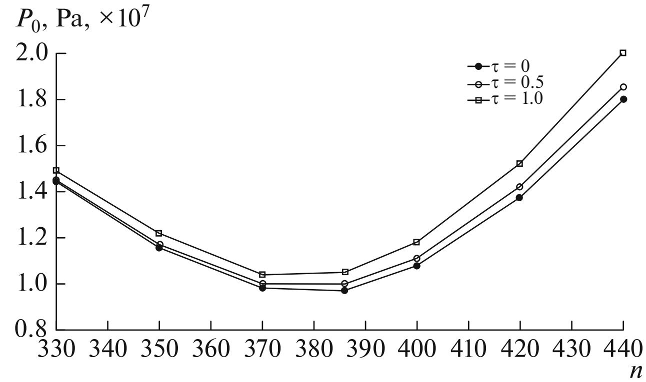

Fig. 3. First zone of instability of a drop surface in parameters (P_0, n) at different moments of slow time.

an oscillating bubble) at the center of the drop. We therefore we may assume there is a spherically symmetric source of mass somewhere deep inside the drop. It causes oscillations of the incompressible near-surface layer, but the very motion of the surface and the near-surface layer is distorted, due to the development of instabilities that emerge on the surface and gradually attenuate as the drop moves away from it.

Small disturbances of a general form can be represented as a superposition of spherical harmonics. Disturbances are large-scale at low orders of spherical harmonics n, so the effects of viscosity can be ignored in low-viscosity liquids (including water). Viscosity can play a prominent role at high values of n. The threshold for the parametric excitation of capillary standing waves on the surface of a drop in particular will be nonzero precisely because of viscosity effects.

Our analysis of viscous phenomena was performed in analogy with the approach used in [8] for oscillations of a spherical gas bubble in a liquid. The deviation of a drop's surface from a sphere was decomposed into spherical harmonics: $\xi(t,\theta,\psi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} a_{nm}(t)\,Y_{nm}(\theta,\psi)$. Different components of the expansion of disturbances (different modes) behave independently of one another, so it suffices to consider only one mode of perturbation. Such consideration produces the equation for the amplitude of the mode perturbations $a = a_{nm}$:

$$\ddot{a} + 2\delta_n \dot{a} + \left[\frac{n + 1/2}{R_0}\dot{V} + \Omega_n^2\right] a = 0. \quad (4)$$

Here, $\delta_n = (n-1)(2n+1)\dfrac{\nu}{R_0^2}$ is the damping decrement of the n-th mode of surface waves; ν is the kinematic viscosity; R_0 is the initial radius of the drop (which varies slightly); $\dot{V}$ is the acceleration on the drop's surface, calculated from the dynamics of the acoustic field in the drop; and Ω_n is the natural frequency of free oscillations of the corresponding mode.

The acceleration profile on the drop's surface is harmonic at the initial moment of time, and the problem is reduced to solving an equation of Mathieu type whose zones of instability are described in the literature [9, 10] and shown in Fig. 2.

The shape of the profile is distorted over time, and its amplitude increases. In addition, the more distorted the profile, the greater its amplitude becomes. However, even such an increase does not lower the threshold of instability. On the contrary: It raises it (see Fig. 3), since the curve has the shape of a burst too short to cause an instability. A bipolar burst is less than 10% the length of a full pulse with a duration of 1 μs (i.e., less than 0.1 μs).

CONCLUSIONS

Calculations show the effect of heat conduction means that compared to a linear case, the center of a drop is not heated to more than fractions of a degree, despite a notable increase in dissipation at the center of the drop. At the same time, the nonlinear amplification of peak pressure is quite strong. Negative pressure levels that exceed those of the liquid's tensile strength can be achieved in practicable modes of forming an acoustic fountain in the center of the drops. Analysis shows that the main reason for the explosion of a drop is the rupturing of the liquid in its center (cavitation) under the action of high negative pressures caused by nonlinear acoustic processes. Our results are in agreement with experimental observations of drop explosions, providing a deeper understanding of the processes that accompany ultrasonic atomization.

Analysis of the instability of a drop's surface shows that its amplitude can be increased when it has certain initial parameters. This considerably distorts the surface, which can lead to a further explosion of the drop. The more turbid form of an upper drop before its explosion when filming an acoustic fountain with a high-speed camera can result from microscopic distortions of the drop surface that are not visible to the naked eye. However, light waves scatter and create an optical effect in the form of turbidity: $\Lambda_n = 2\pi R_0/n$; drop radius $R_0 = 0.75$ mm. The most unstable mode number $n = 386$; i.e., $\Lambda_{386} \approx 12$ μm.

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REFERENCES

1. Khokhlova, T.D., Canney, M.S., Khokhlova, V.A., et al., *J. Acoust. Soc. Am.*, 2011, vol. 130, p. 3498.
2. Hoff, L., *Acoustic Characterization of Contrast Agents for Medical Ultrasound Imaging*, Springer, 2001.
3. Vaezy, S., Martin, R., and Crum, L., *Echocardiography*, 2001, vol. 18, no. 4, p. 309.
4. Maxwell, A., Sapozhnikov, O., Bailey, M., et al., *Acoust. Today*, 2012, vol. 8, no. 4, p. 24.
5. Tomita, Y., *Phys. Fluids*, 2014, vol. 26, p. 097105.
6. Simon, J.C., Sapozhnikov, O.A., Khokhlova, V.A., et al., *J. Fluid Mech.*, 2015, vol. 766, p. 129.
7. Sapozhnikov, O.A. and Annenkova, E.A., *Acoust. Phys.*, 2018, vol. 64, no. 3, p. 299.
8. Prosperetti, A., *Q. Appl. Math.*, 1977, vol. 34, p. 339.
9. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Abramowitz, M. and Stegun, I., Eds., Dover, 1965.
10. Vlasov, V.K., Glukhova, M.N., Korolev, L.N., et al., *Vestn. Mosk. Gos. Univ., Ser. 15. Vychisl. Mat. Kibern.*, 1992, no. 1, p. 65.

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