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CLASSICAL PROBLEMS OF LINEAR ACOUSTICS AND WAVE THEORY

Creating a Reference Plane Ultrasonic Wave in a Fluid Using a Plane Piezoelectric Transducer with a Large Wave Dimension

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Abstract—The article discusses the possibility of using a plane piezoelectric transducer with a large wave dimension as the source of a reference plane ultrasonic wave, which can be used to calibrate hydrophones in the megahertz frequency range. In the experiment, the source was a piezoceramic disk with a diameter of 100 mm and a thickness resonance frequency of about 1 MHz. A method was developed for determining the sensitivity of the transducer in the transmit mode by measuring its electrical impedance. A methodology is proposed for finding the parameters of the plane wave component of the emitted acoustic pulse from a known electrical signal on a generator. It is shown that the acoustic pulse profile measured by a calibrated hydrophone near the source agrees well with the theoretically predicted signal.

Keywords: piezoelectric transducer, hydrophone calibration, plane wave mode

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1. INTRODUCTION

The problem of calibrating sources and receivers is conventional for acoustic research, in particular, in ultrasound applications in medicine, as well as in hydro- and aeroacoustics problems. An important characteristic of any electroacoustic transducer is its sensitivity, which is the relationship between its electrical and acoustic signals. For an acoustic receiver (hydrophone or microphone), sensitivity is understood as the ratio of the electric voltage arising on it to the acoustic pressure at the location of this receiver in its absence. Since any receiver has a finite size, its response to an acoustic field depends on the structure of the field at its location. This is why sensitivity is introduced with respect to a wave with a given structure; a plane incident wave is usually used. It is clear from the foregoing that the ability to create a plane wave with known parameters is fundamental in problems of calibrating receivers. Conventionally, the plane-wave mode is created by placing the receiver in the far field of the acoustic source. The disadvantage of this approach is the need for measurements at large distances from the source; i.e., the corresponding settings cannot be compact. In addition, in far field measurements, significant wave attenuation occurs. Lastly, the relationship between the electrical signal at the source and the corresponding acoustic signal at the receiver depends on many parameters, which reduces the calibration accuracy.

Currently, a large number of methods have been developed for calibrating acoustic sensors and finding acoustic field parameters: the reciprocity and self-reciprocity methods [1, 2], a method for determining acoustic power by measuring the radiation force [3], the variable load method [4], etc. Each of the methods has its advantages and disadvantages. One attractive method for calibrating acoustic pressure sensors is to measure in a reference sound field, i.e., in a field with known characteristics. In this case, the main problem is now to develop a device capable of creating the indicated reference field. In this paper, we propose one variant of this calibration method. It is shown that an ultrasonic transducer, the emitting element of which is a flat piezoceramic disk, can be used as the source of the reference field.

If the diameter of a uniformly polarized flat piezoceramic plate is much larger than the wavelength, then when a pulsed electric voltage is applied to its conducting sides, it changes the plate thickness according to the "piston" law: at all points of the plate (with the exception of small areas near the edge), its surface is displaced equally, thereby emitting a plane wave. Owing to this, there is a region of the space near the surface of the transducer in which an acoustic field in the form of a traveling plane wave is realized over a finite time interval [5]. The specified plane wave can be used as the reference field for calibrating acoustic receivers. Note that an acoustic field close to a plane wave can also be created using multielement arrays, including synthesized ones [6, 7]; however, in practice, using single-element emitters is much simpler.

It is convenient to use a large-diameter plane piezoelectric plate not only because it can generate a plane wave, but also because the electrical and acoustic processes in it can be described by a one-dimensional model of an electroacoustic transducer [8, 9]. With this approach, the transducer is described as a six-terminal network. In this model, the voltage U and current I uniquely associated with acoustic variables—acoustic pressure p_1, p_2 and vibrational speed v_1, v_2 on the inner and outer sides of the transducer, respectively. Knowledge of the piezoelectric and mechanical characteristics of the plate and the wave propagation medium makes it possible to calculate the value of the radiated acoustic pressure on the transducer's surface. Because there is a region of the traveling plane wave near the plate surface, the same sound pressure (only delayed in time) will be measured by a sensor placed at any point in this region. This makes it possible to calibrate the acoustic receiver using a piezoelectric transducer emitting a reference acoustic field. Note that implementation of the described approach requires precise knowledge of the parameters of the piezoelectric transducer.

Thus, to calculate the emitted acoustic field, it is necessary to have information about such parameters of the transducer as the piezoelectric and elastic moduli, electromechanical coupling coefficient, dielectric constant, density, and geometric dimensions of the plate. In this paper, we propose an algorithm of actions that allows, based on electrical and acoustic measurements, to specify the characteristics of the transducer and then calculate the acoustic pressure in the plane wave zone by an analytical method. The results were verified by measuring the acoustic hologram of the transducer [10, 11] and the field on the source axis with a calibrated hydrophone.

2. IMPLEMENTATION OF THE PLANE WAVE MODE NEAR A PISTON SOURCE OF FINITE SIZE

We consider a transducer whose active element is a circular plane piezoceramic disk of thickness l and radius R. Quantity l determines the resonance frequency of thickness oscillations of the transducer: $f_0 = c\,/\,(2l)$, where c is the longitudinal wave velocity in the plate. Let us consider plate oscillations in the one-dimensional approximation. The acoustic field created by such a plate with a uniform distribution of the normal vibrational velocity over the surface is found by solving the problem of a harmonic piston source [12].

We introduce the following quantities, which characterize the harmonic acoustic field: $\mathbf{v}' = (v_x', v_y', v_z')$ is the vibrational velocity vector, the Cartesian components of which can be represented in complex form; in particular, $v_z' = \frac{v_z}{2}e^{-i\omega t} + \frac{v_z^*}{2}e^{i\omega t}$, where v_z is the complex amplitude of the normal vibrational velocity component; here it is assumed that the z axis is oriented perpendicular to the source surface. It is also possible to represent in complex form the acoustic pressure: $p' = \frac{p}{2}e^{-i\omega t} + \frac{p^*}{2}e^{i\omega t}$, where p is the complex pressure amplitude. Subscript 1 denotes the backing — the medium with which the inner surface of the piezoelectric plate comes in contact (in the experiment described below, it was air), and subscript 2 denotes the ambient medium (radiation was produced in water). Given that $v_z\big|_{x^2+y^2\le R^2} = v_2 = \text{const}$, $v_z\big|_{x^2+y^2>R^2} = 0$ (circular piston source), from the solution to the wave equation of the problem in the ambient medium, an expression is obtained for the dependence of the complex pressure amplitude on the plate axis of symmetry on distance z to it [12, 13]:

$$p(0,0,z) = \rho_2 c_2 v_2 \left(e^{ik_2 z} - e^{ik_2\sqrt{R^2+z^2}}\right), \tag{1}$$

where ρ_2, c_2 and $k_2 = \omega/c_2$ are the density, sound speed, and wavenumber in the fluid. If the source were infinitely large ($R \to \infty$), then under the assumption of small absorption in expression (1), only the term $\sim e^{ik_2 z}$ would remain, which corresponds to a plane wave [13]. However, with the finite dimensions of the plate, it is impossible to separate the components of the harmonic signal coming from the edges of the source from the plane wave. On the other hand, for pulsed excitation of the transducer, the nonstationary solution for the field, due to the linearity of the problem, reduces to the superposition of solutions for the spectral components. For an ideal lossless and nondispersive medium, we obtain from expression (1) the following analytical solution for the acoustic pressure on the axis:

$$p'(0,0,z,t) = \rho_2 c_2 \left[V_2\left(t - \frac{z}{c_2}\right) - V_2\left(t - \frac{\sqrt{z^2+R^2}}{c_2}\right)\right], \tag{2}$$

where $V_2(t)$ is the surface velocity of the source. From expression (2) it can be seen that at a distance z from the center of the plate, in the case of the onset of excitation of a pulse signal at moment $t = 0$, there is a time interval $\left[z/c_2, \sqrt{z^2+R^2}/c_2\right]$ within which there is a plane wave signal, while the edge wave described by the second term has not yet reached the measurement point. Thus, within the specified interval, a mode of a plane acoustic wave propagation is realized in its pure form. Figure 1a shows the dependence of the duration of the time interval of the plane wave as a function of distance z from the center of the plate for two source radii: 5 and 2.5 cm. The graph clearly shows that in

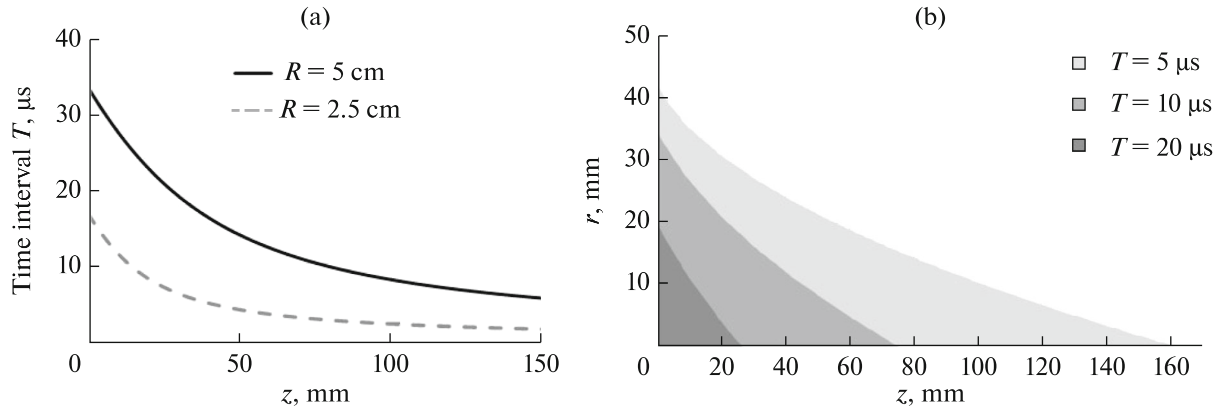

Fig. 1. (a) Estimate of time interval during which plane wave pulse is realized on axis of symmetry of transducers with radii of 5 and 2.5 cm as function of distance to center of transducer; (b) estimate of dimensions of plane wave region in directions along and across beam axis for plane wave intervals of 20, 10, and 5 μs.

order to generate a plane wave, it is necessary to use a transducer with large dimensions, and in this case, measurements must be made near the radiating surface. It is also important to note that the smaller the distance from the piezoelectric plate to the hydrophone, the greater the contribution to the measured signal made by electrical interference from the transducer. Thus, it is important to position the hydrophone optimally to achieve the maximum duration of the plane wave pulse and to avoid electrical interference.

Of interest is also the estimation of the region of existence of the plane wave mode when the hydrophone moves in the direction transverse to the beam axis. The plane wave mode is implemented within the time interval between the moment of arrival of the first signal (the signal of the "direct" wave propagating perpendicular to the surface of the emitter) and the moment of arrival of the signal from the nearest point on the edge of the emitter. It follows that if $r = \sqrt{x^2 + y^2}$—the distance from the axis—then the plane wave mode will be realized during a time interval with a duration

$$T \leq \frac{\sqrt{z^2 + (R - r)^2} - z}{c_2}. \quad (3)$$

The requirement for realization of the plane wave mode within an interval of a given duration T limits the position of the hydrophone. Figure 1b shows the region of realization of the specified mode, calculated by formula (3) for a transducer with a radius R = 50 mm for three different time intervals T: 5, 10, and 20 μs. It can be seen that the smaller the required time interval, the more extended the region of space in which the plane wave mode is realized.

In practice, in addition to an edge wave, delayed signals appear in the fluid caused by excitation of Lamb waves in the piezoelectric plate [14, 15]. Lamb waves are generated at the edge of the piezoelectric plate, where the condition of uniformity of the plate is violated. Propagating towards the center of the plate, they emit an additional signal into the fluid, which can be even stronger than the edge wave. However, like an edge wave, this signal is delayed with respect to the direct wave and therefore does not hinder the formation of the plane-wave mode.

It is noteworthy that such factors as the finite bandwidth of the signal and the diffusion of the waveform with time lead to the fact that the signal duration at the measurement point can exceed the width of the specified time interval of the plane wave.

To verify the above considerations, a number of acoustic signal measurements near the surface of the transducer were carried out with a calibrated hydrophone. The piezoceramic disk had a diameter of 100 mm, its thickness was about 2 mm, and the resonance frequency was 1.12 MHz. An HGL-0200 hydrophone (Onda, USA) was used, the axis of which was oriented perpendicular to the plane of the transducer surface. The hydrophone was located on the axis of symmetry of the source at various distances from its center. A short pulsed electrical signal with a given shape was applied to the transducer. As an example, Fig. 2 shows records of a signal received by a hydrophone at distances of 10, 50, and 100 mm from the transducer. As can be seen from the graphs, at a short distance, the plane and edge waves are almost completely separated, and with distance from the transducer, a zone appears where they overlap. It is also seen that the shape and amplitude of the plane wave component remain unchanged at different distances from the transducer surface, which corresponds to the nature of the plane wave. Therefore, the signal on the surface of the transducer will have precisely the same plane wave component.

A plane wave with constant amplitude and shape was experimentally recorded. Thus, if we determine

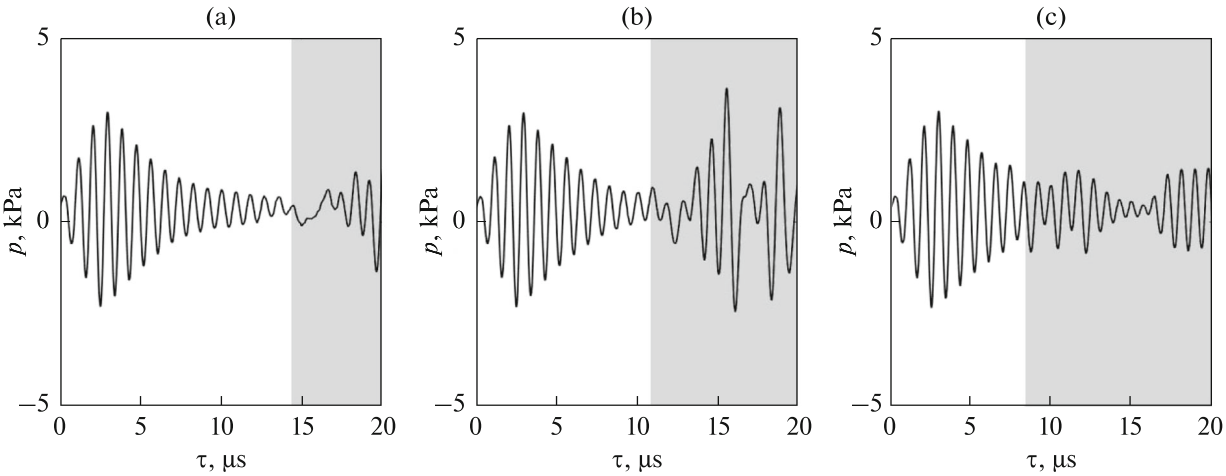

Fig. 2. Sweep of signals in coordinates of traveling wave $\tau = t - z/c_2$ recorded by hydrophone on emitter axis at distances of (a) 10, (b) 50, and (c) 100 mm. Gray color indicates region of recording of edge wave.

the relationship between the electrical signal supplied from the generator to the transducer and the plane acoustic wave emitted by it, we can use the flat transducer as a device for calibrating hydrophones or other transducers in the receiving mode. In this case, calibration using generation of a plane wave does not imply precise location of the measuring device—it is sufficient to place it in the region of generation of the plane wave and the perpendicular to the emitted surface [16].

3. ONE-DIMENSIONAL MODEL OF A PIEZOELECTRIC TRANSDUCER

The electrical signal supplied to the piezoelectric transducer differs in shape and duration from the acoustic signal emitted into the ambient medium. The amplitude of the acoustic signal depends linearly on the amplitude of the electrical signal, as well as on the features of electroacoustic conversion. As the parameter determining the relationship between electrical and acoustic signals, the sensitivity of the transducer in the transmit mode can be chosen, G. The expression for sensitivity can be obtained by considering the process of electroacoustic conversion in the one-dimensional approximation and in harmonic mode.

From the literature [8, 9] it is known how to relate the pressures p_1, p_2 and vibrational velocities v_1, v_2 on the inner and outer surfaces of the piezoceramic plate to the electrical parameters—the voltage on the U_0 and power of the flowing current I_0. Such a relationship can be written in matrix form, which allows the transducer to be represented as a six-terminal network:

$$\begin{pmatrix} p_1 \\ p_2 \\ U_0 \end{pmatrix} = i \begin{pmatrix} \tilde{z}\,\mathrm{cotan}\,kl & -\tilde{z}/\sin kl & h/\omega S \\ \tilde{z}/\sin kl & -\tilde{z}/\mathrm{cotan}\,kl & h/\omega S \\ h/\omega & -h/\omega & 1/\omega C_0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ I_0 \end{pmatrix}, \quad (4)$$

where ω is the angular frequency of harmonic excitation of the system, $\tilde{z} = \rho c$ is the impedance of the piezoceramic plate, $k = \omega/c$ is the wavenumber, ρ and c are the density and sound speed in the piezoelectric plate material, S is the surface area of the transducer, C_0 is the capacitance of the clamped transducer, and h is a coefficient describing electroacoustic conversion. Quantities C_0 and h are related to the electromechanical coupling coefficient k_T, the z component of the piezoelectric modulus e_{z3} and dielectric constant of the blocked transducer ε as follows:

$$C_0 = \varepsilon\varepsilon_0 S/l, \quad k_T = e_{z3}\Big/\sqrt{\varepsilon\varepsilon_0\rho c^2}, \quad h = e_{z3}/\varepsilon\varepsilon_0, \quad (5)$$

where $\varepsilon_0 \approx 8.854\times10^{-12}$ F/m is the electric constant.

The elements of the columns in formula (4) are the complex amplitudes of the corresponding quantities, and it is assumed that the total quantities vary in harmonic law as $\sim\exp(-i\omega t)$. The system of equations (4) makes it possible to calculate the operation of a piezoelectric transducer both in the transmit and receive modes. We are interested in the transmit mode. We assume that the acoustic impedances z_1, z_2 on the inner and outer sides of the piezoelectric plate are known:

$$p_1/v_1 = -z_1, \quad p_2/v_2 = -z_2. \quad (6)$$

Taking into account (6), system of equations (4) takes the form

$$\begin{cases} p_1 = -i\dfrac{\tilde{z}}{z_1}\mathrm{cotan}kl\; p_1 - i\dfrac{\tilde{z}}{z_2}(1/\sin kl)\; p_2 + i\left(h/S\omega\right) I_0, \\ p_2 = -i\dfrac{\tilde{z}}{z_1}\dfrac{1}{\sin kl}\, p_1 - i\dfrac{\tilde{z}}{z_2}\mathrm{cotan}\,kl p_2 + i\left(h/S\omega\right) I_0, \\ U = -i\left(h/\omega\right)\dfrac{p_1}{z_1} - i\left(h/\omega\right)\dfrac{p_2}{z_2} + i\dfrac{1}{\omega C_0} I_0. \end{cases} \quad (7)$$

From the first and second equations of system (7), we express p_1 and p_2 through the current I_0:

$$p_1 = \frac{ih}{S\omega} \frac{\sin kl + i\dfrac{\tilde{z}}{z_2}(\cos kl - 1)}{\left(1 + \dfrac{\tilde{z}^2}{z_1 z_2}\right)\sin kl + i\tilde{z}\left(\dfrac{1}{z_1} + \dfrac{1}{z_2}\right)\cos kl} I_0,$$
$$p_2 = \frac{ih}{S\omega} \frac{\sin kl + i\dfrac{\tilde{z}}{z_1}(\cos kl - 1)}{\left(1 + \dfrac{\tilde{z}^2}{z_1 z_2}\right)\sin kl + i\tilde{z}\left(\dfrac{1}{z_1} + \dfrac{1}{z_2}\right)\cos kl} I_0. \quad (8)$$

Substituting these expressions in the third equation of system (7) and taking into account formulas (5), we obtain the following relationship between the complex amplitudes of the voltage across the transducer and corresponding current:

$$\frac{U_0}{I_0} = Z_0 = \frac{1}{-i\omega C_0} \times \left[1 - \frac{k_T^2}{kl} \frac{i\dfrac{z_1 + z_2}{\tilde{z}}\sin kl + 2(1 - \cos kl)}{\left(1 + \dfrac{z_1 z_2}{\tilde{z}^2}\right)\sin kl + i\dfrac{z_1 + z_2}{\tilde{z}}\cos kl}\right]. \quad (9)$$

This ratio of the complex amplitudes of voltage and current is the electrical impedance of the element Z_0. Note that the voltage U_0 and current I_0 in formula (9) corresponds to the values directly on the piezoelectric conducting plates. In practice, to supply a signal to the piezoelectric plate, an electric cable is used, one end of which is connected to the conducting sides of the piezoelectric plate, while at the second end there is a connector to which the generator supplies electric voltage. Therefore, the voltage and current at the input to the transducer are the voltage U and current I at the end of the cable, which differ from the corresponding values directly on the piezoelectric plate, U_0 and I_0. Taking into account the influence of the cable, the expression for the transducer's impedance $Z = U/I$ is as follows [17]:

$$Z = R_c \frac{Z_0 - iR_c \tan(\omega L/v_c)}{R_c - iZ_0 \tan(\omega L/v_c)}, \quad (10)$$

where R_c is the wave impedance of the cable, L is its length, v_c is the velocity of electromagnetic wave propagating in the cable. When deriving relation (10), it was taken into account that the electrical cable is a transmission line, i.e., a waveguide for electromagnetic waves. The cable properties are determined by its length L, the propagation velocity of electromagnetic waves in it v_c, and the wave impedance R_c.

From (10), it is clear that if the transducer were a matched load with an impedance $Z_0 = R_c$, then the impedance Z would be equal to Z_0 regardless of the cable length. This is generally not the case. For example, if the cable is short (its length is much shorter than the length of the electromagnetic wave), then the tangent in formula (10) can be replaced with its argument, which gives $Z = (Z_0 - i\omega \mathcal{L}_c L)/(1 - i\omega C_c L Z_0)$, where $\mathcal{L}_c = R_c/v_c$ and $C_c = 1/(R_c v_c)$ have the meaning of the linear inductance and linear capacitance of the cable, respectively. The typical values of these quantities for a 50-Ω cable are $\mathcal{L}_c = 0.275\ \mu\text{H/m}$ and $C_c = 110\ \text{pF/m}$ [18]. At low load $Z_0 \ll R_c$ a short cable behaves like an inductance $Z \approx -i\omega L\mathcal{L}_c$, and under heavy load $Z_0 \gg R_c$ like capacitance $Z \approx i/(\omega L C_c)$. As a rule, however, an intermediate case is realized, therefore, for a correct description, it is necessary to use the more general formula (10).

Cable output voltage U due to voltage on the piezoelectric plates U_0 in the following way:

$$\frac{U}{U_0} = \cos(\omega L/v_c) - i\frac{R_c}{Z_0}\sin(\omega L/v_c). \quad (11)$$

The ration $G = p_2/U$ can be considered the transfer function of piezoelectric conversion in the emission mode. The expression for it follows from formulas (8) and (11):

$$G = \frac{ih}{S\omega} \frac{\sin kl + i\dfrac{\tilde{z}}{z_1}(\cos kl - 1)}{\left(1 + \dfrac{\tilde{z}^2}{z_1 z_2}\right)\sin kl + i\tilde{z}\left(\dfrac{1}{z_1} + \dfrac{1}{z_2}\right)\cos kl} \times \frac{1}{Z_0 \cos(\omega L/v_c) - iR_c \sin(\omega L/v_c)}. \quad (12)$$

In practice, it is not always possible to work with harmonic signals and, in addition, each transducer takes some time to establish a stationary harmonic signal emitted into the ambient medium, which may not be sufficient due to the narrowness of the time window in the plane wave mode in an experiment. Therefore, it is proposed to work with short pulsed signals and use the above theory for the harmonic mode for the spectral components of the emitted pulsed signal of interest to us, which can be found using Fourier transform: $\tilde{p}_2(t) \xleftrightarrow{\mathcal{F}} p_2(\omega)$, $\tilde{U}(t) \xleftrightarrow{\mathcal{F}} U(\omega)$, where $\tilde{p}_2(t)$ and $\tilde{U}(t)$ are time-dependent signals whose spectral amplitudes are $p_2(\omega)$ and $U(\omega)$, respectively.

Thus, the six-terminal model establishes the relationship between the voltage on the generator and the acoustic pressure on its surface. If the transducer parameters are known, then the characteristics of the radiated sound wave can be immediately determined from the electrical signal supplied to the transducer. Indeed, if we measure the parameters of the indicated electrical signal, then we can calculate the time dependence of the acoustic pressure on the surface of the emitter in a one-dimensional approximation and therefore completely describe the plane-wave component of the radiated acoustic field (see Section 2). Thus, it becomes possible to create a reference plane wave, which can be used to calibrate hydrophones by comparing the electrical signal of the receiver and a given acoustic signal measured in the plane-wave mode.

4. DETERMINING THE UNKNOWN TRANSDUCER PARAMETERS BY MEASURING THE ELECTRICAL IMPEDANCE

In practice, often, not all transducer parameters are known, or they are known with limited accuracy, and to obtain reliable results, these constants should be specified. In this case, an analytical expression for the electrical impedance of the piezoelectric transducer (10) is useful. The impedance is easily measured in the required frequency range [19, 20] using only an oscilloscope and a resistor with a known nominal value; the obtained frequency dependence contains all the necessary constants and parameters of the piezoceramic plate. Since the number of measurement points can a priori be made larger than the number of unknown parameters, these parameters can be found by minimizing the standard deviation of the theoretical dependence on the experimental one. After specifying the necessary plate constants, it becomes possible to estimate the acoustic pressure on the transducer surface.

The obtained frequency dependence of the electrical impedance (10) includes the parameters of the transducer that may be unknown in advance. Let us suppose that the geometric dimensions of the plate (thickness l, area S) and density ρ, which for the piezoelectric plate of the used type PZT-4 is $\rho = 7560$ kg/m^3, are known, and the remaining parameters are set only with limited accuracy. Then the unknowns are the capacitance of the clamped transducer C_0, electromechanical coupling coefficient k_T, and longitudinal wave velocity in the piezoelectric c, which enters into the expression for the wavenumber and acoustic impedance.

To determine the transducer parameters, the following sequence of actions is proposed. The first step is to experimentally measure the frequency dependence $Z(\omega)$ in water and air. Measurements should be carried out in a wide frequency range, decreasing the step near the resonance frequencies. Some of the unknown parameters can be determined from the experimental frequency dependence of the impedance without additional calculations. Next, it is necessary to choose the unknown parameters numerically, so that the theoretical dependence of the impedance on frequency (10) best coincides with the experimentally measured dependence. The following describes the sequence of actions that must be performed to determine all unknown parameters of the transducer based on the experimental measurement of its electrical impedance.

Step 1. Finding Resonance Frequency of the Transducer

When the experimental measurements are taken $Z(\omega)$ in water and air, the resonance frequency of the transducer can be found from the maximum of the real part of the impedance. Measurements in water and air, as well as the wide frequency range that envelops both the first and third resonances, make it possible to find the resonance frequency of the piezoelectric plate with a certain accuracy. In our experiments, it was $f_0 = 1.120 \pm 0.002$ MHz and the error was estimated by comparing the resonance frequencies of the first, third, and fifth harmonics. The resonance frequency makes it possible to calculate the unknown ratio of the longitudinal wave velocity to the plate thickness by formula $c/l = 2f_0$.

Step 2. Determining the Electrical Capacitance of the Piezoelectric Plate

At this stage, it is possible to estimate the capacitance of the transducer C_0. The method considers the low-frequency asymptotics of the impedance, $\underset{\omega\to 0}{Z_0} \approx \frac{i}{\omega C_0}$. If we measure the impedance of the transducer in the low-frequency region, then from the approximation of the dependence of the imaginary part of the impedance on the frequency as a hyperbola, it is possible to estimate the sought capacitance (Fig. 3a). Since the low-frequency asymptotics depends only on the capacitance of the piezoelectric plate, the dependences of the imaginary part of the impedance on frequency during measurements in water and air should be identical, which is observed in experiment. In practice, the hyperbolic dependence is distorted due to the occurrence of Lamb waves, but the distortion decreases with increasing frequency. The found capacitance of the piezoelectric plate was $C_0 = 21.195 \pm 0.004$ nF; the error was estimated by comparing independent measurements in water and air. Knowing the geometric dimensions of the plate, from the approximation of the low-frequency asymptotics, the dielectric constant ε can be estimated quite accurately, which in the future will help with more accurate selection of the remaining transducer characteristics.

Step 3. Determining the Parameters of the Electric Cable Connected to the Transducer

The influence of the cable and the need to take it into account when measuring the impedance is shown in Fig. 3b: the electrical cable changes the slope of the imaginary part of the transducer, which is particularly

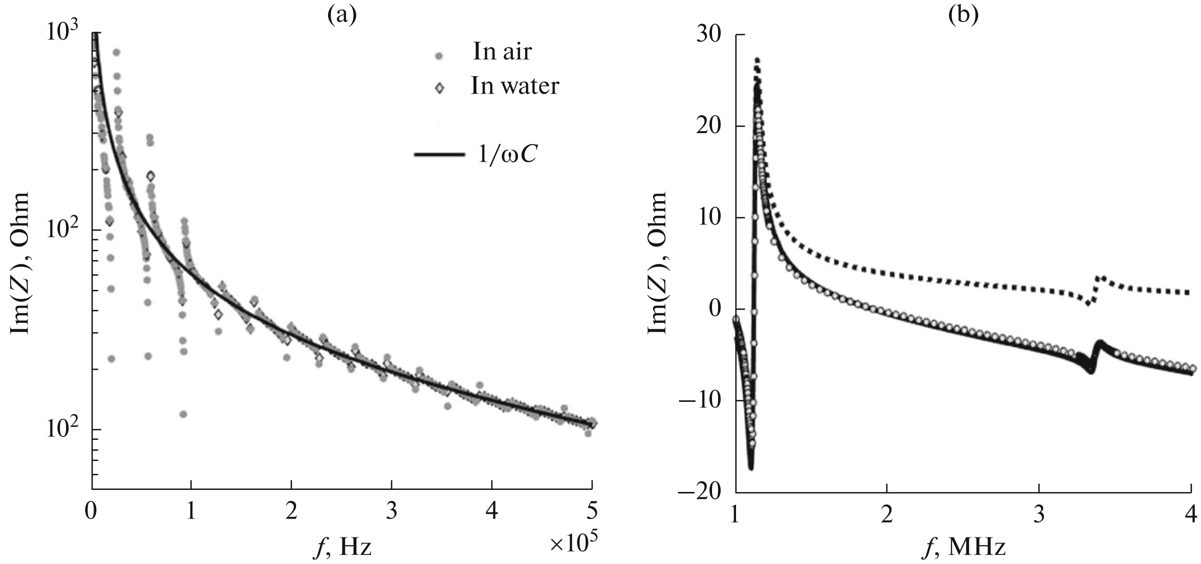

Fig. 3. (a) Approximation of imaginary part of experimentally measured impedance by hyperbolic dependence in low-frequency region to determine capacitance of clamped transducer, (b) comparison of experimentally measured frequency dependence of imaginary part of transducer impedance (circles) with theoretical formula for Z_0 disregarding influence of electric cable (dashed line) and dependence for Z taking into account cable (solid line).

significant in the frequency range between f_0 and $3f_0$. The dependence of the imaginary part of the impedance $\mathrm{Im}(Z)$ on frequency in this region is mainly due to the hyperbolic asymptotics ~ $\frac{1}{\omega C_0}$ and influence of the cable. Quantity $\mathrm{Im}(Z)$ is virtually independent of the acoustic impedance of the ceramics $\tilde{z}$ and coefficient k_T, since the resonance factor is relatively small in this region. Therefore, setting tabular (i.e., approximate) values for unknown quantity k_T and using the found parameters C_0 and c/l, by comparing the experimentally measured imaginary part of the impedance with formula (10), we can determine the phase delay introduced by the cable, i.e., the ratio of its length to the speed of electromagnetic waves L/v_c. The experiment used an electrical cable with a wave impedance $R_c = 50\ \Omega$. Since it is easy to measure the length of the electrical cable, to check the correctness of the calculation procedure, it is possible to calculate the velocity of electromagnetic waves and compare the result with the tabulated values. In our experiment, the cable length was $L = 1.00 \pm 0.001$ m and the found phase delay turned out to be equal $L/v_c = (6.359 \pm 0.002) \times 10^{-9}$ s, which corresponded well to the typical values for the velocity of electromagnetic waves in the cable [18]. The error for L/v_c was also estimated by comparing measurements of this quantity when the transducer was immersed in water and air. In the experiments, the measurement results in these two cases did not differ within the above error.

Step 4. Determining the Electromechanical Coupling Coefficient k_T from Impedance Measurements in Air

From the formula for the impedance of the piezoelectric plate (9), it can be seen that it includes the ratios of the acoustic impedances of the media z_1 and z_2 to the impedance of the ceramic $\tilde{z}$. If the air is on the left and right of the piezoelectric plate, then this ratio becomes small. Therefore, formula (9) can be transformed as follows:

$$Z_0\big|_{z_1, z_2 \to 0} = \frac{1}{-i\omega C_0}\left[1 - \frac{k_T^2}{kl}\frac{2(1-\cos kl)}{\sin kl}\right]. \tag{13}$$

In this case, at the intercept of the imaginary part of the transducer impedance of the abscissa axis, we can find the unknown electromechanical coupling coefficient k_T [9]:

$$k_T^2 = \frac{\frac{\pi}{2} f_1/f_0}{\tan\left(\frac{\pi}{2} f_1/f_0\right)}, \tag{14}$$

where the frequency f_1 corresponds to equality of the imaginary part of the impedance of the transducer to zero.

Since the experiment measures the transducer impedance taking into account the cable, to estimate the impedance of the piezoelectric plate itself and find k_T, the influence of the cable needs to be eliminated. Using the found value L/v_c, we transform formula (10) and obtain the following expression for the impedance of the plate:

Table 1. Found transducer parameters

R, mm	ρ, kg/m^3	f_0, MHz	C_0, nF	L/v_c, 10^{-9} s	k_T	tan δ, 10^{-3}
50	7560	1.120 ± 0.002	21.19 ± 0.04	6.359 ± 0.002	0.463 ± 0.004	4.92 ± 0.08

$$Z_0^{\exp} = R_c \frac{Z_{\exp} + iR_c \tan(\omega L/v_c)}{R_c + iZ_{\exp} \tan(\omega L/v_c)}. \quad (15)$$

Now it remains only to determine at what frequency the imaginary part of the impedance of the transducer in air intersects the abscissa axis and calculate the electromechanical coupling coefficient. In our experiment, these values were $f_1 = 1.014 \pm 0.001$ MHz and $k_T = 0.462 \pm 0.004$, and the error f_1 corresponded to the frequency step when measuring the electrical impedance of the transducer, and the error k_T was calculated as indirect depending on the error of quantities f_0 and f_1.

Step 5. Determining the Loss Tangent of the Piezoceramics

The last unknown parameter is the tangent of the angle of mechanical and electrical losses tan δ, which contributes to the imaginary part of the wave vector as follows: $k = \frac{\omega}{c}(1 + i\tan\delta)$. Its value can be determined by the peak value of the real part of the impedance in resonance regions. It is more correct to search for the loss coefficient tan δ in air, because in water there are additional losses associated with radiation. However, in both air and water, at the first resonance, the influence of Lamb waves, which substantially distort the shape of the impedance dependence (Figs. 4a, 4c), is strong. Therefore, for an air load, it is advisable to search for the loss tangent at the third and fifth resonance harmonics: the influence of Lamb waves on them is already small (Figs. 4d, 4e). By comparing the experimentally measured real part of the impedance in the regions of resonance frequencies with formula (10), the value of the loss coefficient was found tan δ: at frequencies $3f_0$ and $5f_0$ in air, it was equal to $\tan\delta = (4.92 \pm 0.08) \times 10^{-3}$, where the error was estimated by comparing measurements in two different frequency domains. Within the specified error, the loss tangent tan δ did not change with frequency. This value tan δ also satisfies measurements in water at a frequency $3f_0$ (Fig. 4b). Therefore, the found loss coefficient can be applied in the entire considered frequency range.

Thus, after performing the above five steps in the expression for impedance (10), all the sought parameters of the piezoelectric transducer are determined. Constant h, according to (4), is found from the above quantities—the dielectric constant ε entering into the transducer capacitance C_0 and the electromechanical coupling coefficient k_T.

The results of comparing the experimental data with the theoretical dependence for water and air are shown in Figs. 5a, 5b. As a result of applying the described algorithm of actions, the parameters of the piezoceramic transducer used in the study were determined. They are given in Table 1.

5. MEASURING THE ACOUSTIC PRESSURE IN THE PLANE WAVE MODE AND COMPARISON WITH THE CALCULATION IN THE ONE-DIMENSIONAL TRANSDUCER APPROXIMATION

During the experiment (Fig. 6), a known electrical signal $\tilde{U}(t)$ was supplied from the frequency generator to the transducer, which in turn emitted an acoustic wave consisting of plane and edge components. At a certain distance from the transducer, in the region where these components were separated in time, a hydrophone was placed and the received signal was recorded. The plane wave component $\tilde{p}_2(t)$, corresponding to the calculated time window in the frequency space is related to $U(\omega)$ by (12). Thus, the experimentally measured signal $p^{\exp}(t)$ and theoretical signal $\tilde{p}_2(t)$ calculated from the known set voltage on the generator $\tilde{U}(t)$ were compared. Note that the calculated signal $\tilde{p}_2(t)$ corresponds only to the plane wave emitted by the transducer, since the proposed theory does not take into account edge effects and surface waves.

The signals were emitted by a transducer, which was a circular piezoceramic plate with a diameter of 100 mm and a thickness of 2 mm, mounted in a sealed metal case with rings of conductive rubber located on opposite sides of the piezoelectric plate near its edge. Each of these rings was formed by a closed rubber cord with a circular cross section having a diameter of 1 mm, which made it possible to ensure electrical contact with the silvered surface of the plate with minimal mechanical impact on its vibration. Because the back load of the piezoceramic plate was air, the transducer had a high mechanical Q-factor ($Q \sim 10$); therefore, the duration of the emitted acoustic wave turned out to be several times longer than the period of the electric signal set by the generator. The construction of the transducer is described in more detail in [5].

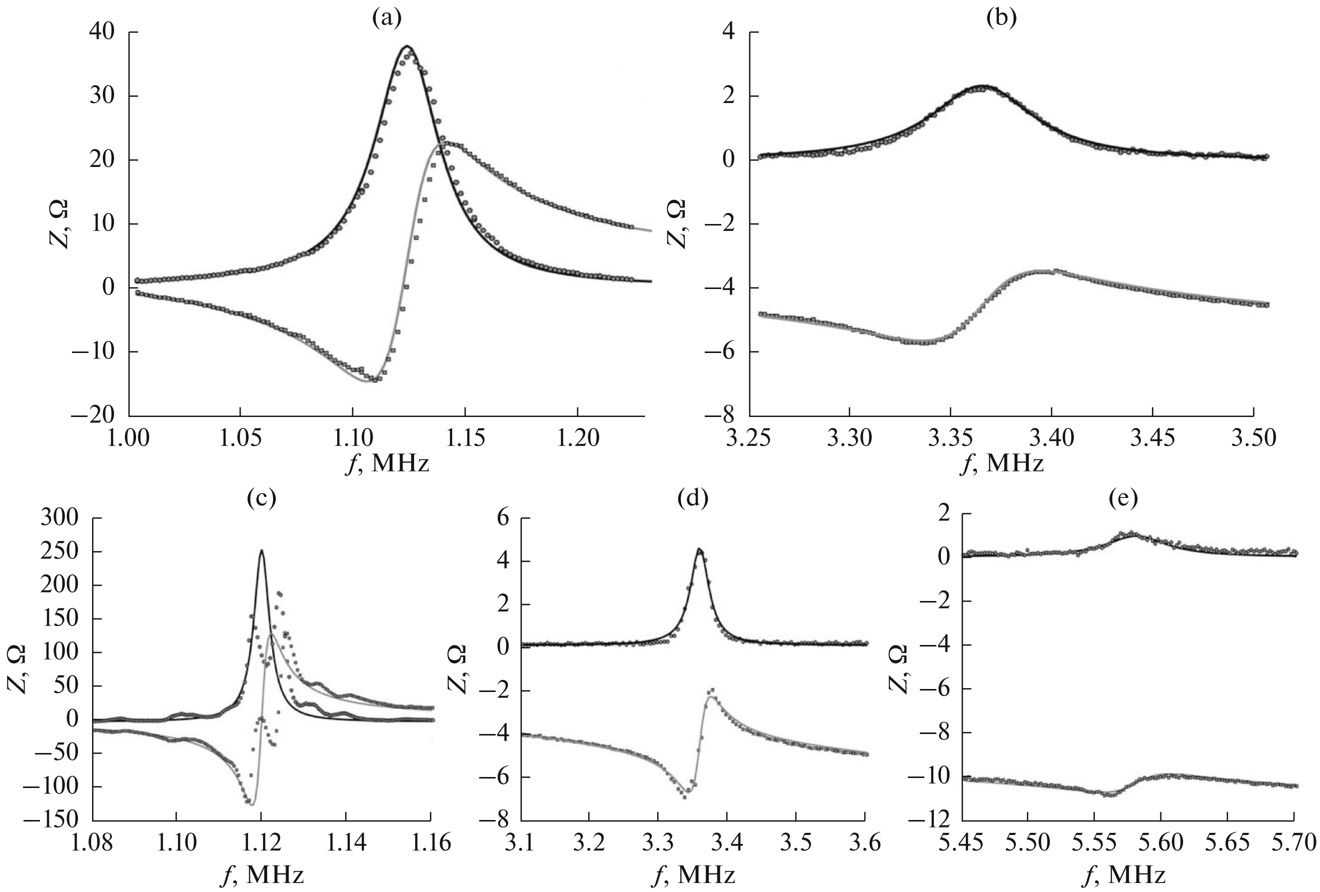

Fig. 4. Comparison of imaginary and real parts of impedance experimentally measured (squares and circles, respectively) and theoretically calculated (gray and black solid lines, respectively) for measurements (a, b) in water and (c–e) in air at resonance frequencies.

The electrical signal was formed by an Agilent 33250A generator. The amplitude of the pulses supplied to the transducer was 3 V. As pulses, signals consisting of one or three sine periods at the transducer resonance frequency were used. To record a signal in the region of realization of a plane wave, the above-mentioned calibrated HGL-0200 hydrophone (Onda, USA) was used. The factory sensitivity of the hydrophone was specified by the manufacturer with an accuracy of 1 dB; within the error in the studied frequency band, it was –262 dB to V/μPa at 1 MHz.

Figure 7 shows the records of the electric signal on the transducer (Figs. 7a, 7b) and the corresponding acoustic signals (Figs. 7c, 7d). The experimentally measured signals, taking into account the sensitivity of the hydrophone, were compared with acoustic signals theoretically calculated by formula (12), where the electrical signal spectrum U corresponded to the records in Figs. 7a, 7b. The measurements showed that for both transducers, the calculated signal describes very well the measured signal, both its shape and amplitude, fitting the factory sensitivity error of 1 dB.

When deriving (12), which relates the acoustic pressure to the voltage on the generator, no restrictions on the signal duration were used. Thus, for operation of the transducer in quasi-harmonic mode, it is also possible to estimate the amplitude of the emitted har-

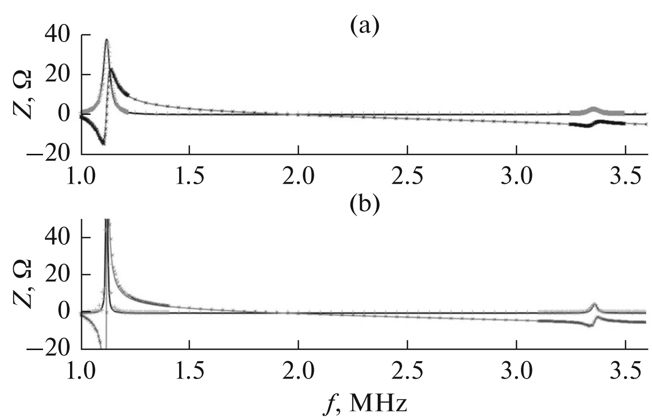

Fig. 5. Comparison of imaginary and real parts of impedance experimentally measured (squares and circles, respectively) and theoretically calculated (gray and black solid lines, respectively) for measurements in (a) water and (b) air in wide frequency range.

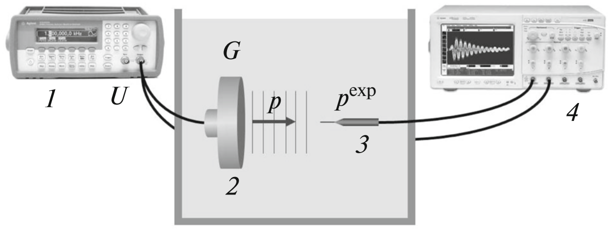

Fig. 6. Diagram of experiment on emission and reception of plane wave. *1*, generator; *2*, piezoelectric transducer; *3*, hydrophone; *4*, oscilloscope.

monic wave. Figure 8 compares the calculated acoustic pressure for a plane wave with the experimentally measured pressure when an electric signal was applied to the transducer for a duration of 60 sine periods at the transducer resonance frequency. With a sufficiently large diameter of the transducer and, consequently, a large time window corresponding to the plane wave propagation mode, the amplitude of the received signal has time to stabilize at a constant level, and it can easily be compared with the calculated one in the region where the oscillation amplitude is constant over several periods. The oscillations that occur afterwards, corresponding to the arrival of edge waves, also stabilize after some time, and the total pressure amplitude at the measurement point can be found by comparing the amplitudes of the established plane wave and total fields.

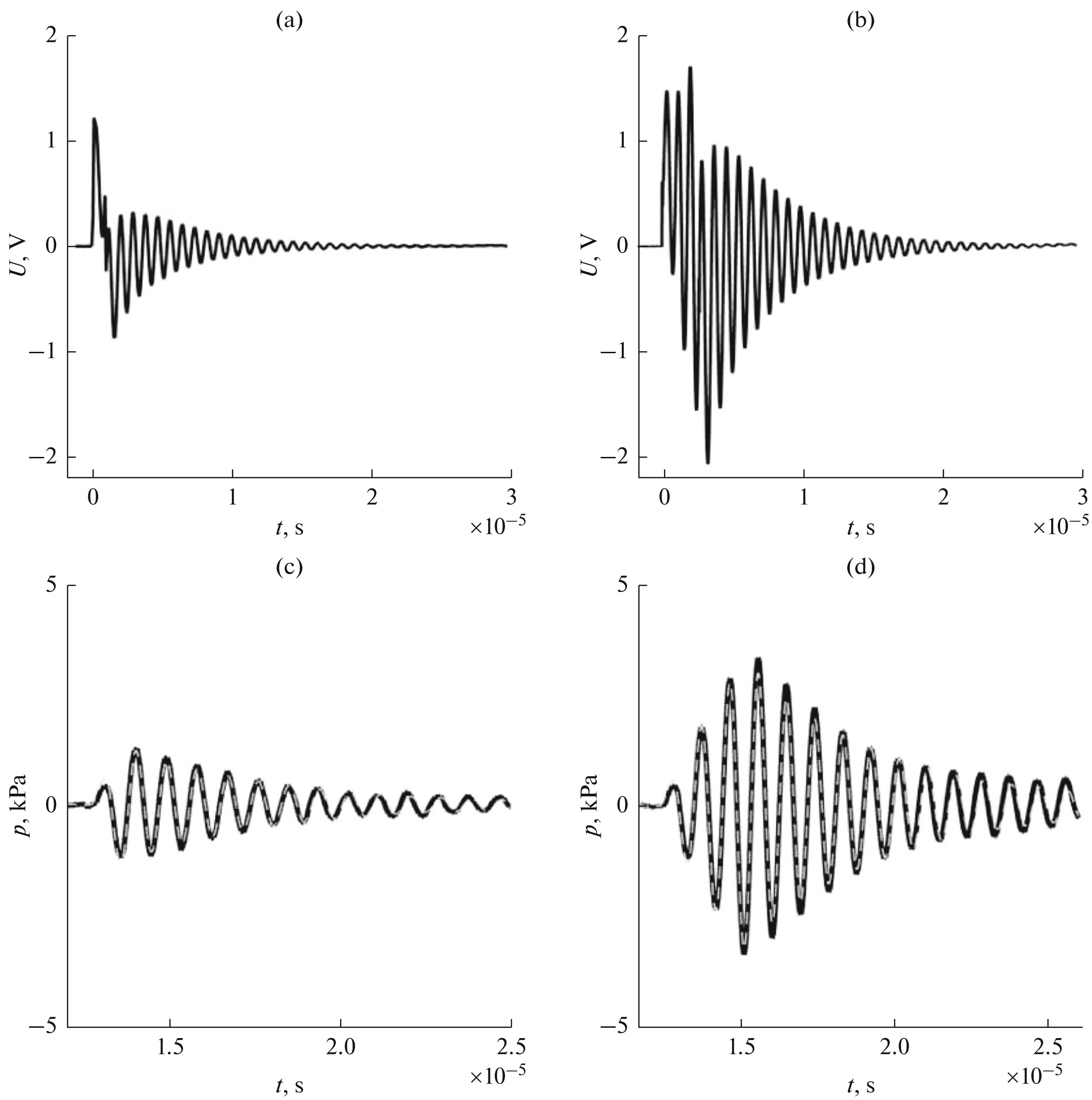

Fig. 7. Electrical signal measured at the transducer with the shape of the electrical signal at the generator in the form of (a) one sine period at resonance frequency and (b) three sine periods at resonance frequency; (c, d) comparison of corresponding experimentally measured pressure signal (black line) with theoretically calculated one (gray line).

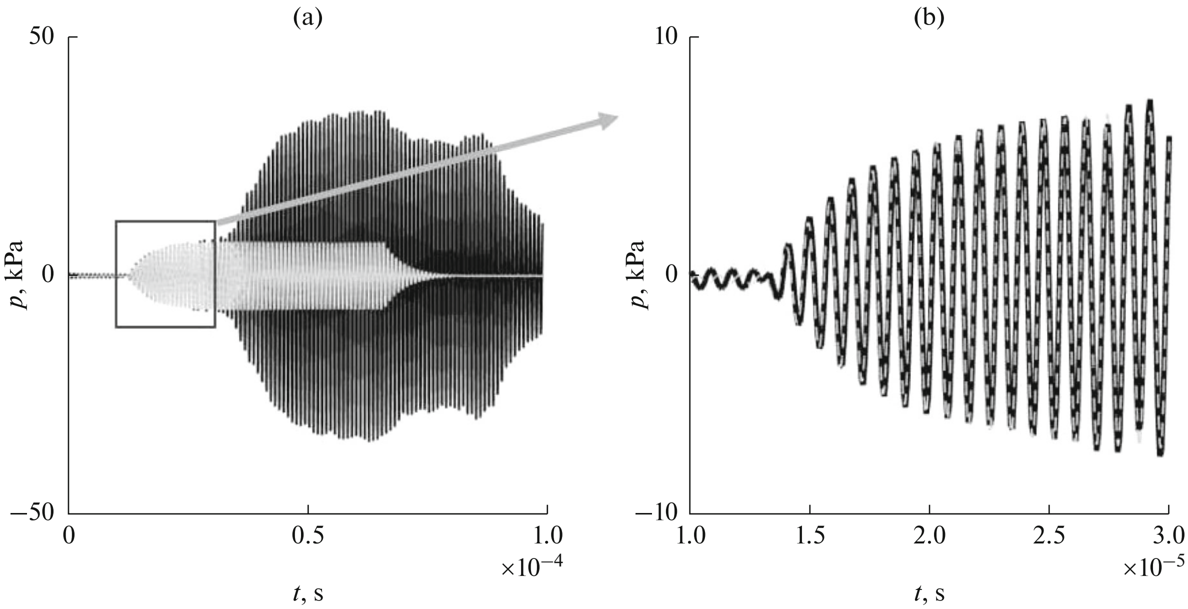

Fig. 8. Comparison of experimentally measured signal (black line) with theoretically calculated one (gray line) for quasi-harmonic electrical signal (60 sine periods at resonance frequency set on generator).

When the transducer is excited in a continuous sinusoidal signal mode, we are interested in the relationship between the amplitudes of the emitted acoustic wave p_2 and electrical voltage at the input of the transducer U. Figure 9 shows the frequency dependence of the sensitivity calculated in by formula (12) in emission mode $|G| = |p_2/U|$ for the studied piezoelectric transducers. The indicated dependence makes it possible to determine the emitted acoustic pressure from the electric voltage measured on the emitter, i.e., to use the transducer as a reference source in some finite frequency range. A natural limitation in forming the desired emitted signals is the resonance nature of the transducer's sensitivity. Due to the high mechanical Q-factor, the spectrum of the pulsed acoustic signal is rather narrow and, therefore, the amplitudes of the experimentally measured and theoretically calculated signals can be compared only for spectral components near the resonance frequency. To calibrate the hydrophone in a broad frequency band, it is necessary to either use a transducer with broad resonance (this is possible via correct selection of the backing), or several transducers with different resonance frequencies.

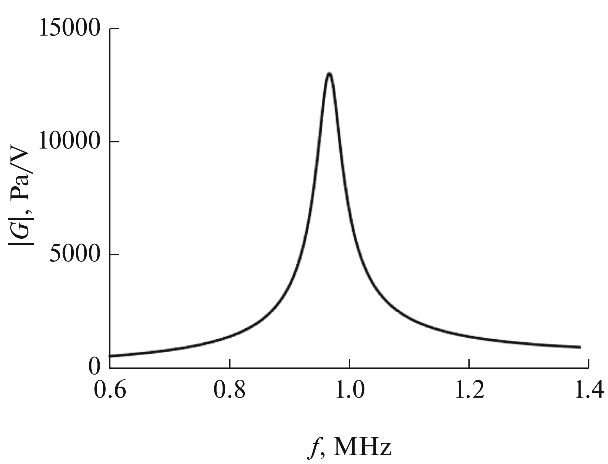

Fig. 9. Ratio of pressure emitted by transducer to voltage on transducer as function of signal frequency.

Thus, the proposed method makes it possible to use a plane piezoelectric transducer with a large wave dimension to emit a plane acoustic wave of known amplitude and shape. This feature is attractive for calibrating hydrophones [21–24].

6. CONCLUSIONS

The study demonstrated the possibility of using a plane piezoelectric transducer with a large wave dimension as the source of a plane wave field with a known temporal profile. In the case when the characteristics of the transducer for some reason are unknown, a method has been proposed for determining these characteristics by measuring the electrical impedance.

The use of a piezoelectric transducer as the source of the sound field of a known plane wave is possible in a wide frequency range, since it is always possible to choose a piezoelectric plate whose thickness resonance corresponds to the required frequency interval. The use of a matched backing and matching layers, as well as the use of gradient piezoceramic plates [25], when the piezoelectric characteristics vary linearly in thickness, as well as ultrasonic piezoelectric transducers with controlled characteristics [26], makes it possible to smooth the resonant peaks and increase the operating frequency range. Thus, the stated results and methods are a good basis for calibrating hydrophones and calculating the amplitude of the emitted fields.

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REFERENCES

1. R. J. Bobber, *Underwater Electroacoustic Measurements* (Naval Res. Lab. Washington, 1970; Mir, Moscow, 1974).
2. V. K. Dolya, Soviet Physics Acoustics. **33** (4), 367 (1987).
3. A. V. Nikolaeva, S. A. Tsysar', and O. A. Sapozhnikov, Acoust. Phys. **62** (1), 38 (2016).
4. A. G. Sanin, P. K. Chichagov, and A. M. Reiman, in *Ultrasonic Diagnostics. Collection of Scientific Papers of Institute of Applied Physics of RAS* (Gorkii, 1983), p. 21 [in Russian].
5. M. V. Khasanova, S. A. Tsysar', D. A. Nikolaev, and O. A. Sapozhnikov, Memoirs of the Faculty of Physics, Lomonosov Moscow State University, No. 5, 1750709-1 (2017).
6. D. Nikolaev, S. Tsysar, A. Krendeleva, O. Sapozhnikov, and V. Khokhlova, Proc. Meet. Acoust. **38**, 045012/1 (2019).
7. A. E. Isaev, Yu. M. Aivazyan, and A. M. Polikarpov, Al'm. Sovr. Metrol., No. 1, 163 (2020).
8. A. A. Kharkevich, *The Theory of Electroacoustical Transformers. Selected Works in Three Vols.* (Nauka, Moscow, 1973), **Vol. 1** [in Russian].
9. G. S. Kino, *Acoustic Waves: Devices, Imaging and Analog Signal Processing* (Prentice-Hall, Englewood Cliffs, NJ, 1987; Mir, Moscow, 1990).
10. O. A. Sapozhnikov, Yu. A. Pishchal'nikov, and A. V. Morozov, Acoust. Phys. **49** (3), 354 (2003).
11. O. A. Sapozhnikov, S. A. Tsysar, V. A. Khokhlova, and W. Kreider, J. Acoust. Soc. Am. **138** (3), 1515 (2015).
12. L. F. Lependin, *Acoustics* (Vysshaya Shkola, Moscow, 1978) [in Russian].
13. V. V. Krylov, *Foundation of Sound Radiation and Scattering Theory* (Moscow Univ., Moscow, 1989) [in Russian].
14. D. Cathignol, O. A. Sapozhnikov, and J. Zhang, J. Acoust. Soc. Am. **101** (3), 1286 (1997).
15. O. A. Sapozhnikov and M. A. Smagin, Acoust. Phys. **61** (2), 181 (2015).
16. A. A. Dorofeeva and O. A. Sapozhnikov, Memoirs of the Faculty of Physics, Lomonosov Moscow State University, No. 5, 1750301-1 (2017).
17. A. D. Grigor'ev, *Electrodynamics and Microwave Frequency Engineering* (Vysshaya shkola, Moscow, 1990) [in Russian].
18. K. J. Parker and E. M. Friets, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **34** (4), 454 (1987).
19. A. V. Egorov, I. V. Ovchinnikov, and I. A. Zhukov, Izv. Altai. Gos. Univ. **1** (1), 125 (2010).
20. V. S. Kononenko, Acoust. Phys. **52** (6), 696 (2006).
21. D. R. Bacon, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **35** (2), 152 (1988).
22. M. E. Schafer, in *Ultrasonic Exposimetry,* Ed. by M. C. Ziskin and P. A. Lewin (CRC Press, Boca Raton, FL, 1993), Chap. 8.
23. C. Koch and W. Molkenstruck, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **46** (5), 1303–1314 (1999).
24. A. E. Isaev and I. V. Chernikov, Acoust. Phys. **61** (6), 699 (2015).
25. D. Yamazaki, K. Yamada, and K. Nakamura, Jpn. J. Appl. Phys. **40** (12), 7166 (2001).
26. V. V. Kazakov and A. G. Sanin, Acoust. Phys. **63** (1), 104 (2017).