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PHYSICAL ACOUSTICS

Compression and Amplification of an Ultrasonic Pulse Reflected from a One-Dimensional Layered Structure

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Abstract—Compression of ultrasonic pulses reflected from layered structures is studied. A short pulse is emitted into water towards a structure consisting of solid plates backed with an air layer. Due to multiple reflections in the structure, the signal is elongated. The reflected signal is received by the same transducer and digitized. After that, the wave is reversed in time and emitted towards the layered structure for the second time; then, the reflected signal is received. Due to the invariance of the processes under the time reversal, the pulse is compressed by the structure: the reflected signal becomes shorter and acquires the waveform of the initial pulse. The possibility of an efficient compression of signals is demonstrated experimentally. Numerical simulations show that the use of more complex structures can considerably increase the compression ratio and produce short signals of a much higher amplitude than that emitted by the transducer. An efficient compression algorithm is proposed.

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INTRODUCTION

In engineering and medical applications of ultrasound, it is often necessary to generate high-amplitude ultrasonic waves. Because the maximum acoustic pressure on the surface of an actual acoustic source is limited, high-intensity fields are usually produced by spatial focusing. A concave shape of the transducer or acoustic lenses or mirrors create such conditions that ultrasonic energy distributed over a large area becomes concentrated in a small (focal) region. The wave is thereby amplified in the sense that its amplitude noticeably increases. Clearly, the total energy of the wave does not change, it is only redistributed in space. The wave focusing is used in acoustics and optics, its efficiency being evident. However, the potentialities of the passive wave amplification are not limited to spatial focusing. Formally, the four-dimensional space–time domain, in which the wave process is analyzed, allows focusing not only in the spatial coordinates, but also in time. By analogy with focusing in space, which collects the wave energy from a large region into a small one, focusing in time collects the energy from a long time interval into a short one. In other words, it is necessary to somehow compress the wave signal in time. Work in this direction was started not long ago. This approach was most thoroughly developed in optics, where laser pulses were compressed by transmitting frequency modulated signals through a dispersive medium, which was represented by optical fiber or by a diffraction grating [1]. In acoustics, investigations in this direction are as yet rare. Note that signals may be compressed not only for their amplification, but also for increasing the resolution of diagnostic systems.

The time compression of acoustic signals is closely related to the new method in acoustics, namely, the time reversal of waves [2–4]. The progress in this area is primarily due to M. Fink and his disciples, who proposed and implemented the *time-reversal mirror* [2]. At present, such a mirror is implemented as a two-dimensional multielement array of broadband transceiving transducers. The transducers are controlled by an electronic device, which records the entire waveforms received by the transducers, reverses these signals in time, amplifies them, and applies them again to the same transducers for transmitting. Clearly, this time-reversal mirror reflects the wave with a certain delay, because the signal must be recorded within a certain time interval. This electronic device is an important component of the wave reversal system. The development of a time-reversal mirror based on other principles, for example, on nonlinear wave effects in the material of the mirror, has as yet been successful only for sinusoidal waves [5], although research into the time reversal of nonharmonic waves is also being conducted [6]. In the case of sinusoidal waves, their time reversal is equivalent to wave front reversal, which is thoroughly studied in optics [7, 8].

Systems that rely on the time reversal principle compensate for spatial and temporal distortions of a wave that passes through a lossless inhomogeneous medium. Applications of this method are numerous: problems of underwater acoustics [9, 10], nondestructive testing [11], and ultrasonic surgery [12, 13] are among them. However, in this paper, we focus on another feature offered by the time reversal of waves: the possibility to compress and amplify signals. The idea of signal compression through the time reversal is rather simple. Let a short acoustic pulse pass through an inhomogeneous layer. Due to multiple reflections, the pulse not only suffers spatial distortions, but also becomes longer. Let this long signal be recorded, reversed in time, and emitted back into the medium. Then, after it passes through the inhomogeneous layer again, the signal must become shorter and acquire the waveform of the original pulse. In this case, the role of the compression device is played by the passive inhomogeneous layer. Note, however, that this layer compresses not an arbitrary signal, but an appropriately chosen one. An interesting illustration of this principle can be found in [14], where an *ultrasonic bazooka*, i.e., a device that produces an intense pulse focused at a given point by transmitting the radiation emitted by a multielement source through a pipe whose walls provide multiple reflections, is described.

The use of an arbitrary inhomogeneous medium can produce a shorter pulse, but such a system may be not the optimal one. An efficient compression system must employ a specially tailored inhomogeneity. This aspect of systems based on the time reversal of waves is studied in the present paper.

PLANE-LAYERED MEDIUM FOR PULSE COMPRESSION

The use of multielement sensors considerably complicates time reversal systems. At the same time, many effects, including the pulse compression, can be realized with one transducer if the wave propagates in one dimension. (Compression with one transducer is basically possible when the wave propagation is not one-dimensional, but this approach is difficult to implement.) One-dimensional waves are observed in the field produced by a planar transducer whose dimensions are much greater than the wavelength. In the near-field zone of the transducer, diffraction effects can be neglected and the wave can be considered as a plane wave. For this wave to remain a plane wave as it propagates through an inhomogeneous medium, the medium must be one-dimensional, i.e., plane-layered. As the inhomogeneous medium, our experimental study employs a structure consisting of one or several plane-parallel solid plates immersed in water. This system is characterized by high dispersion, which distorts the signals propagating in it. Because the coefficient of ultrasonic absorption in the plates and water is low, the energy loss in the reflected wave is small.

To compress the waveform, the reflection from each of the individual plates that compose the reflecting inhomogeneous medium must be weak. Indeed, this condition is necessary for the set of plates to convert a short incident pulse into a long reflected signal of a smaller amplitude and, after the time reversal, to convert the long pulse into a short high-amplitude pulse. Weakly reflecting plates may be produced in two ways. The first way is to use thin plates, i.e., plates whose thickness is much smaller than the acoustic wavelength. The reflection coefficient of a thin layer is a linear function of its thickness; i.e., it may be arbitrarily small. The second way is to use resonant plates, i.e., plates whose thickness is an integer multiple of the half-wavelength in the material of the plate. Such plates are known to totally transmit the incident wave at the resonant frequency; i.e., the reflection coefficient of a harmonic signal is zero. By detuning the plate from its resonance or by increasing the bandwidth of the signal, one obtains a desired reflection coefficient.

In the megahertz frequency range, which we use in our study, the thin plates must be several hundredth of a millimeter thick and several centimeters in diameter. It is therefore impossible to use glass, while metal layers (foil) are difficult to make plane and parallel. The situation with resonant plates is much more favorable: the thickness of half-wavelength plates is on the order of one millimeter. The single disadvantage of the resonant plates when compared to the thin plates is that the low reflection coefficient can only be achieved with narrowband signals, i.e., radio pulses. These plates cannot be used to compress video pulses. Therefore, the experiments described below were performed with radio pulses reflected from a stack of resonant plates.

EXPERIMENTAL SETUP

The schematic diagram of the setup is shown in Fig. 1. The acoustic pulses were emitted and received by the same circular planar transducer 25 mm in diameter (Panametrics V194). The transducer operated in the frequency band from 3 to 6 MHz. Electric voltage was supplied to the transducer from a broadband signal source that was capable of synthesizing arbitrary waveforms using up to 65536 samples within a specified time interval (Agilent 33250A). The acoustic signal reflected from the inhomogeneous layer was received by the transducer and applied to a digital oscilloscope (Tektronix TDS 520A). The digitized signal was then fed into a computer through a GPIB bus and was appropriately processed, in particular, time reversed. The computer also controlled the signal source.

In the experiments, we also used relatively broadband signals. It was therefore necessary to take into account the transducer's frequency response. To this end, we measured the amplitude of a harmonic wave reflected from a thick aluminum plate as a function of frequency. To measure this characteristic, we excited the transducer by a rectangular-envelope radio pulse at a given frequency. The duration of the signal was chosen such that it was long enough for the amplitude to reach the steady state and also such that the radiation terminated by the time the return signal arrived. We measured the amplitudes of the incident and reflected pulses on the intervals where the envelope reached its steady state (was constant). The frequency response (on receive and transmit) obtained in this way was used in calculations and to compare the original signal with that after two reflections. In addition to the frequency-dependent effect on the signal amplitude, the electric circuit and acoustic transducer also introduced a frequency-dependent phase shift. However, the additional phase advance φ occurred in the experiments twice: when the original signal was transmitted and received and when the time-reversed reflected signal was transmitted and received. Therefore, the phase advance introduced into the original signal, after the signal was reversed in time, became equal to –φ and completely compensated for the second phase advance. Thus, the frequency dependence of the phase had no effect on the signal reflected twice.

The reflecting structure was fabricated from artificial sapphire plates characterized by the velocity of longitudinal waves c = 11080 m/s and by the density ρ = 3850 kg/m^3. Note that, due to the high velocity and acoustic impedance of sapphire, the frequency range, in which reflection from the plates is small, is narrow; i.e., it is necessary to work with relatively narrowband signals. However, when our experiments were performed with aluminum, which is more suitable in this sense (c = 6320 m/s and ρ = 2700 kg/m^3), we found that the reflecting structure did not possess the property of invariance under time reversal. The possible origin of this effect may be the loss due to the scattering of the ultrasonic wave from the granular aluminum structure or the fact that the plates were insufficiently plane and homogenous in thickness. The sapphire plates were lossless and, to a high accuracy, plane-parallel. They were about 1 mm thick. To provide the total reflection, the bottom plate was loaded by an air layer (Fig. 1). The planes were adjusted to be parallel to the working surface of the transducer using special leveling screws. The adjustment procedure started with the plate that was farthest from the transducer; the criterion for the parallelism was maximization of the amplitude of the reflected signal. The experiment used ultrasonic signals with a center frequency of about 5.5 MHz, which satisfied the condition of half-wavelength resonance in the plates.

PULSE COMPRESSION USING A ONE-LAYER REFLECTOR

Invariance of the processes studied in the experiment with respect to the time reversal procedure was first examined using a reflector consisting of one plate. With this configuration, a low loss could be achieved in the experiment and simple formulas could be used in the theory.

First, let us theoretically consider the process of reflection from one plate. Let a wave be incident from the medium with an acoustic impedance z_0 onto the plate with a thickness h and an impedance z. Behind the

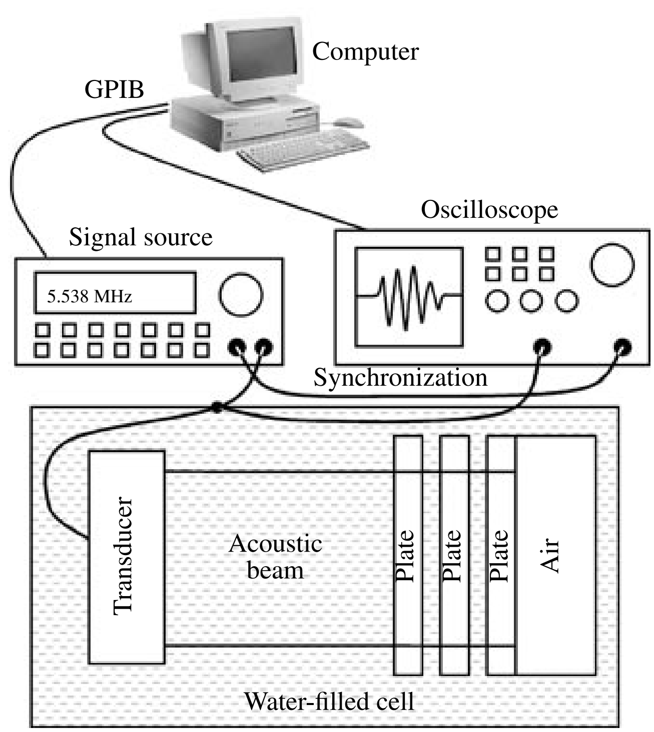

Fig. 1. Schematic diagram of the experimental setup.

plate, there is the third medium with an impedance z_{load}. In our study, the first medium is water, the second medium is sapphire, and the third medium is air. The coefficient of reflection of a harmonic wave from the plate is given by the formula [15]

$$R(\omega) = \\ = \left(R_0 + R_{load}\exp\left(2i\frac{\omega}{c}h\right)\right)\Big/\left(1 + R_0R_{load}\exp\left(2i\frac{\omega}{c}h\right)\right), \quad (1)$$

where $R_0 = (z - z_0)/(z + z_0)$ and $R_{load} = (z_{load} - z)/(z_{load} + z)$ are the coefficients of reflection from the interfaces of the respective half-spaces. For the water-to-sapphire interface, the reflection coefficient is close to 1 ($R_0 \approx$ 0.932); therefore, broadband short signals are almost totally reflected from this interface. However, for sufficiently long radio pulses, the amplitude of the reflected wave may be reduced due to the interference of waves reflected from the plate's surfaces. Denote the incident and reflected waveforms as $p_{inc}(t)$ and $p_{ref}(t)$. To be specific, consider the Gaussian pulse

$$p_{inc}(t) = A_{inc}\exp\left(-\frac{t^2}{\tau^2}\right)\sin(2\pi f_0 t). \quad (2)$$

Let the plate be half-wavelength thick at the central frequency of the signal: $f_0 = c/(2h)$. Let us introduce the quantity

$$Q = f_0\tau, \quad (3)$$

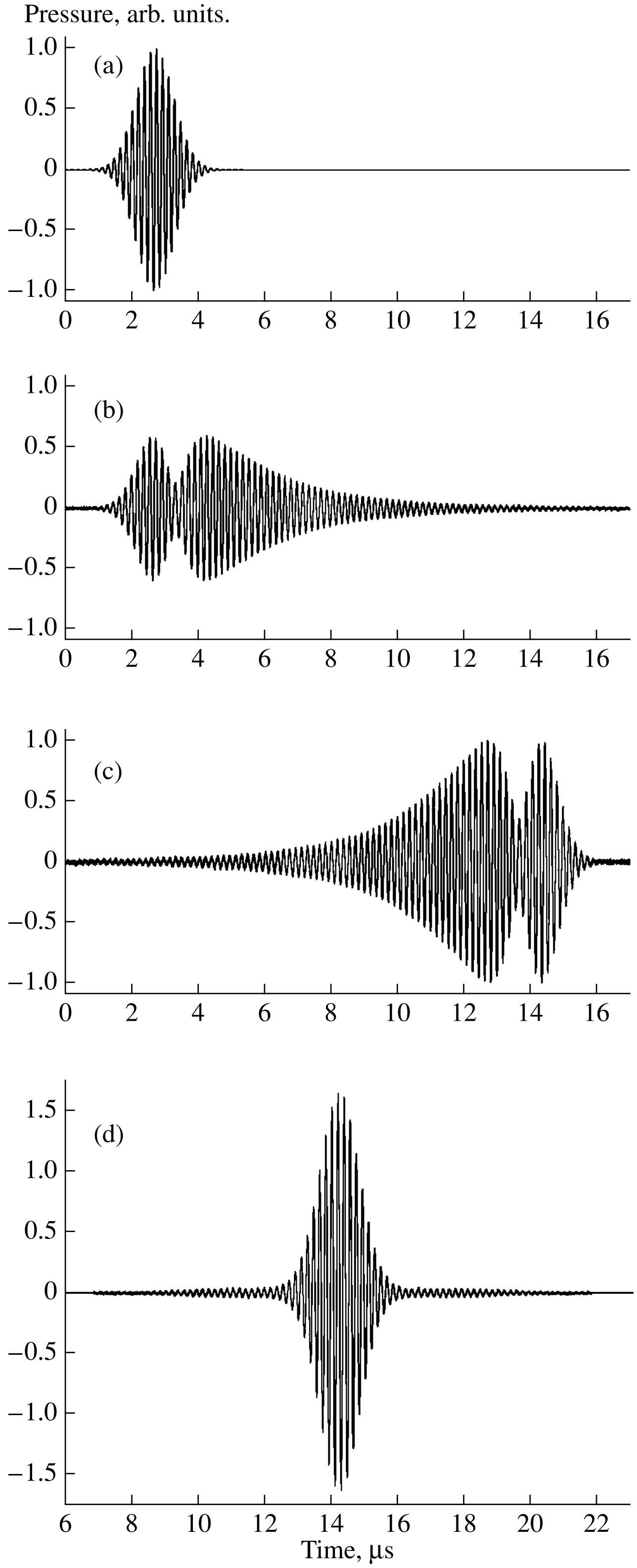

Fig. 2. Experimental results for the one-layer structure: (a) initial pulse, (b) pulse obtained as a result of reflection of the initial pulse, (c) pulse obtained from the reflected pulse by time reversal and amplification to the level of the initial pulse, and (d) compressed pulse obtained as a result of reflection of the time-reversed pulse.

which has the meaning of the number of periods that fit within half of the pulse duration. We consider the case important for applications in which the plate is acoustically loaded by a softer medium (e.g., water or air): $z_{load} \ll z$. In this case, $R_{load} \approx -1$ and the following approximate analytical solution can be obtained for the reflected waveform:

$$p_{ref}(t) \approx A_{inc}\left[R_0 e^{-\frac{t^2}{\tau^2}} - \frac{(1-R_0^2)Q\sqrt{\pi}}{2R_0}\exp[(Q\ln(-R_0R_{load}))^2/4] \times\left\{1+\operatorname{erf}\left[\frac{t}{\tau}+\frac{Q\ln(-R_0R_{load})}{2}\right]\right\} \times\exp\left[Q\ln(-R_0R_{load})\frac{t}{\tau}\right]\right]\sin(2\pi f_0 t), \quad (4)$$

where $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-\zeta^2}d\zeta$ is the error integral. The analysis of expression (4) shows that the envelope $|p_{ref}(t)|$ is a double-humped function with two maxima separated by an interval of about 2τ wide and with a long tail, which decays as $\exp\left[Q\ln(-R_0R_{load})\frac{t}{\tau}\right]$. For the optimal compression, it is necessary that the amplitudes of the maxima be equal. For the air load ($R_{load} = -1$), formula (4) imposes the following condition on the number of periods in the initial Gaussian pulse: $Q \approx 3.9$. To choose the signal parameters more accurately, we used a numerical technique based on the spectral approach (see below) and obtained the refined value of $Q \approx 4.4$.

In the experiment, the transducer emitted the Gaussian pulse given by Eq. (2) with the parameters $f_0 = 5.5$ MHz and $\tau = 0.8$ μs ($Q = 4.4$). The distance to the reflector was 14 cm, so that the reflected signal arrived at the transducer after the emission process was terminated. The results are shown in Fig. 2. Figure 2a displays the initial Gaussian pulse, and Fig. 2b, the reflected pulse. The reflected waveform is in good agreement with that predicted by approximate Eq. (4). As will be shown below, more accurate calculations give almost perfect agreement with the experiment. The reflected signal is by a factor of approximately 1.6 smaller in amplitude than the initial pulse and is almost 3 times longer.

The received reflected signal was digitized, time reversed, and increased in amplitude to the level of the initial signal (Fig. 2c). After that, the signal obtained was emitted again and received by the transducer after the second reflection from the plate. The result with allowance for two multiplications by the system's frequency response is plotted in Fig. 2d. It can be seen that the signal has become shorter and its waveform proves to be identical to that of the initial Gaussian pulse. Thus, we managed to compress the pulse while completely reconstructing its waveform and to increase its peak pressure by a factor of 1.6. Consequently, the wave processes in the system under study were indeed invariant with respect to the change of sign of the time variable.

To independently verify the results of acoustic measurements of the signal waveforms, we performed an additional experiment using optical visualization of the pulses under study by the shadowgraph method (Toepler method) [16]. To reduce optical aberrations, a parallel beam of light was formed and focused with the help of wide-aperture parabolic mirrors. As a source of light, we used a diode laser, which produced short 10- to 100-ns-long flashes synchronized with the ultrasonic signal. During such a short illumination time, the acoustic wave traveled a distance much smaller than the wavelength; therefore, the shadowgraph image mapped a frozen picture of the ultrasonic field. This image was observed with the help of a digital video camera. Figure 3 shows two such pictures. Figure 3a is a shadowgraph image of the long double-humped pulse that propagates from top downwards to the reflector (at the bottom). One can clearly see not only the region occupied by the ultrasonic signal, but also individual wave fronts. Expectedly, the fronts have the form of straight segments; i.e., the wave is a plane wave within the beam boundaries. The theoretical spatial distribution of acoustic pressure is shown in the right-hand panel. The lower plot shows the same signal after its reflection from the sapphire plate loaded by air. Now, the signal propagates upwards. The main result is that the reflected pulse really became shorter, as predicted by the theory (the waveform on the right-hand side). Thus, the fact of signal compression as a result of reflection from the one-layer structure is corroborated.

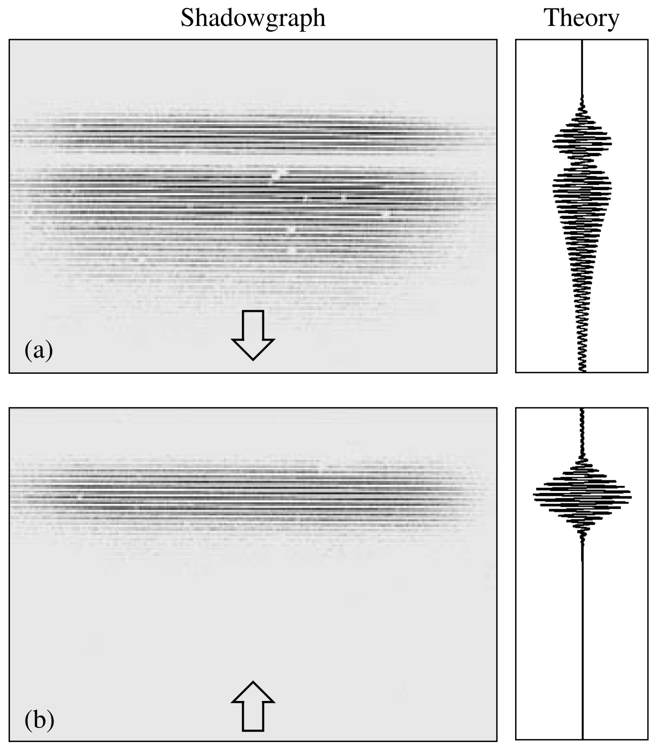

Fig. 3. Shadowgraphs of the pulse obtained in the compression experiment with the one-layer structure: (a) before the reflection and (b) after the reflection. The arrows show the pulse propagation direction; the reflector is at the bottom of the figure. The panels at the right show the theoretical waveforms of the respective pulses.

THEORETICAL MODEL OF REFLECTION FROM A MULTILAYER STRUCTURE

The above analysis of reflection from one plate has shown that, even in this simple case, to find the waveform of the reflected wave, it is necessary to sum infinite series; representation in terms of analytical functions is possible only approximately. At the same time, the spectral description proves to be simpler because, for a harmonic wave, its waveform does not change as it propagates in the layered structure and, therefore, the process of reflection can be characterized by the reflection coefficient $R(\omega)$ for the complex amplitude. For one plate, the reflection coefficient is given by Eq. (1). For a greater number of layers, the solution can be constructed using the transfer matrix method [17]. The general procedure for calculating the reflected waveform has the standard form: it begins with calculation of the incident spectrum, then calculates the reflected spectrum using the known $R(\omega)$, and finally performs the inverse Fourier transform to calculate the reflected waveform.

Consider the process of harmonic wave propagation in the multilayer medium following the transfer matrix method. Let n be the layer index and P_n^+ and P_n^- be the complex amplitudes of the waves that propagate rightwards and leftwards, respectively (Fig. 4). The amplitudes are specified at the left boundary of the layer. The amplitudes in two adjacent layers are related through the matrix equation

$$\begin{pmatrix} P_{n-1}^+ \\ P_{n-1}^- \end{pmatrix} = \hat{M}_{n-1,n} \begin{pmatrix} P_n^+ \\ P_n^- \end{pmatrix},$$

$$\hat{M}_{n-1,n} = \begin{pmatrix} \frac{z_n + z_{n-1}}{2z_n}\exp(-ik_n h_n) & \frac{z_n - z_{n-1}}{2z_n}\exp(ik_n h_n) \\ \frac{z_n - z_{n-1}}{2z_n}\exp(-ik_n h_n) & \frac{z_n + z_{n-1}}{2z_n}\exp(ik_n h_n) \end{pmatrix}, \quad (5)$$

where $\hat{M}_{n-1,n}$ is the transfer matrix, which depends on the acoustic impedances z_n, velocities of sound c_n, and

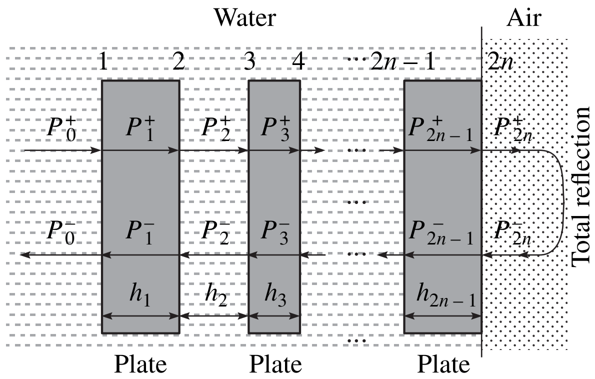

Fig. 4. Schematic diagram of the multilayer reflecting structure and notations used in the numerical simulations.

layer thicknesses h_n; $k_n = \omega/c_n$ is the wave number; and ω is the circular frequency. Equations (5) written for all of the layers yield the following matrix expression, which relates the incident and reflected waves in water (P_0^+ and P_0^-) to the incident and reflected waves behind the stack of plates:

$$\begin{pmatrix} P_0^+ \\ P_0^- \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} P_N^+ \\ P_N^- \end{pmatrix}, \qquad (6)$$

where the total transfer matrix $\hat{M} = \|m_{ij}\|$ is calculated by successively multiplying the transfer matrices of all the layers: $\hat{M} = \hat{M}_{0.1}\hat{M}_{1.2}\ldots\hat{M}_{N-2,N-1}\hat{M}_{N-1,N}$. Note that $N = 2N_*$, where N_* is the total number of the plates, is an even number. Let $R_{load} = P_N^-/P_N^+ = (z_{load} - z_N)/(z_{load} + z_N)$ be the coefficient of reflection from the last interface and z_{load} be the acoustic impedance of the medium to the right of the last layer. Expression (6) yields the desired reflection coefficient of the layered structure: $R = P_0^-/P_0^+ = (m_{21} + R_{load}m_{22})/(m_{11} + R_{load}m_{12})$. In particular, in the case of the air load $z_{load} \approx 0$ that we study, we obtain $R_{load} = -1$, which yields $R = (m_{21} - m_{22})/(m_{11} - m_{12})$. To adjust the system, we also loaded the last layer with water or a thick metal layer. In this case, the calculations were performed using the appropriate acoustic impedances z_{load}.

In the computer implementation of the above algorithm, the signals were specified on a finite time interval, whose length was long enough for the transients to terminate; i.e., the signal was close to zero at the ends of the interval. On this interval, the continuous signal was sampled with the sampling interval Δt that was much shorter than the period of the highest frequency used. When changing from signals in the time domain to their spectra and back, we used the fast Fourier transform. The typical values of T and Δt were 500 μs and 10 ns, respectively; the number of plates in the structure was up to $N_* = 15$. The algorithm and related code written in the Delphi language, which were run on a Pentium 4 personal computer, calculated one reflection event in 1 to 2 s.

OPTIMIZATION OF THE COMPRESSION PROCESS

The problem of increasing the amplitude of the reflected wave at the expense of its compression can be formulated as follows: let an acoustic source emit a signal $p_{inc}(t)$ into the medium. The peak pressure in the signal does not exceed a certain value A_{inc}, while the duration τ_{inc} of the signal may be arbitrarily long. It is necessary to design such a plane-layered inhomogeneity that increases the peak pressure of the reflected signal $p_{ref}(t)$ to a specified value A_{ref} and reduces its duration to τ_{ref}. Basically, the solution to the problem is easy to find if the system is invariant under the time reversal operation: the desired stratified inhomogeneous layer must be such that, being illuminated with a short signal $p_{ref}(t)$ that has a peak pressure of A_{ref} and duration τ_{ref}, it produces the reflected signal $p_{inc}(t)$ with a peak pressure of no higher than A_{inc}. The lower the amplitude of the reflected signal, the higher the gain $K = A_{ref}/A_{inc}$ due to the time reversal processing. It is clear that the duration τ_{inc} of the signal is of no significance; the main condition is that the signal be recorded within this time. However, this simple solution is no longer valid in practice at long τ_{inc}, because the distance $c\tau_{inc}$ the wave travels through the multilayer structure in the process of reflection also becomes considerable (c is the velocity of the wave). As the distance increases, the energy lost by the wave unavoidably grows and the system loses its property of being invariant under time reversal. Consequently, an additional requirement is that the desired gain K be achieved at the shortest possible duration τ_{inc}.

RESULTS OF SIMULATION

The mathematical model described above was used to design multilayer structures that provide efficient compression. As the signal that was to be obtained as a result of the compression, we took Gaussian radio pulse (2). The duration of the pulse was characterized by the number of periods Q of the fundamental frequency present within the characteristic pulse duration τ (see Eq. (3)). If Q is given, the determination of the layered structure necessary for the compression is reduced to finding the maximum possible amplitude gain and the number of plates required to achieve this gain. The design parameters of the reflecting structure that were to be found were the distances between the plates and their thicknesses. The material was considered to be specified, because a smooth variation of the parameters is difficult to implement in practice.

Note that the structure of the reflected signal is such that its beginning is formed by the layers of the reflecting structure that are nearest to the transducer; more distant layers make their contribution later. Based on this fact, we chose the following structure design algorithm. We emphasize that the steps described below are performed in the framework of the mathematical model.

First, we consider the reflection of the Gaussian signal of amplitude A_0 with the given number of periods Q from the half-wavelength plate loaded (on the right) with water. The reflected signal has a lower amplitude A_1 and a longer duration. Its envelope has the form of a double-humped function and is approximately described by Eq. (4) with $R_{load} = (z_0 - z)/(z_0 + z) = -R_0$. At this stage, we make a conclusion concerning the maximum gain of the structure being designed: it is equal to A_0/A_1.

Next, we load the plate with air and analyze the resulting signal. If this signal has no spikes with the amplitude greater than A_1, we terminate the design procedure, because adding more plates will not improve the structure's property to amplify the time-reversed signal. If high spikes are observed, we make a conclusion that a second plate should be added to suppress them. The second plate is chosen to have the same thickness as the first one and is placed as close as possible to it, but the distance must be no shorter than the length of the original pulse: only in this case the reflection from the second plate will not change the amplitude A_1 of the first reflection. Due to the second plate, the reflected signal will be additionally distorted, but, because its passband is the same as that of the first plate, the amplitude A_2 of the spikes that appear will be smaller than A_1 (otherwise, the plate is too close to the first one; i.e., the gap should be increased). After that, we slightly change, say decrease, the thickness of the plate by keeping its position the same. The plate is thereby detuned from the half-wavelength resonance and, as a consequence, its reflectivity increases. We vary the amplitude until the spike amplitude A_2 increases to the level of the amplitude of the first spike A_1. After that, we load the second plate with air. If no higher-amplitude spikes appear in the resulting waveform after the peak A_2, we terminate the design procedure. Otherwise, it is continued in a similar manner: we add the third plate, and so on.

Note that this algorithm gives a regular technique to choose the parameters of the reflecting structure that provides the desired amplitude amplification of the reflected wave with a small number of plates. In principle, a slightly more compact arrangement can provide the same gain with a smaller number of plates. However, the properties of the structure become very sensitive to the distance between the plates; a very small change in the distance or speed of sound may suppress the amplification properties, which should be avoided in practice.

Figure 5a shows the gain that is calculated by the above algorithm and is the desired as a result of compression versus the number Q of periods in the signal. As can be seen from this figure, the longer the resulting signal, the higher the amplitude gain achieved. As we noted above, this property is associated with the resonance nature of reflection from the half-wavelength plate: the longer the signal, the narrower its bandwidth and the weaker its reflection from an individual plate are; i.e., the longer the signal being compressed may be.

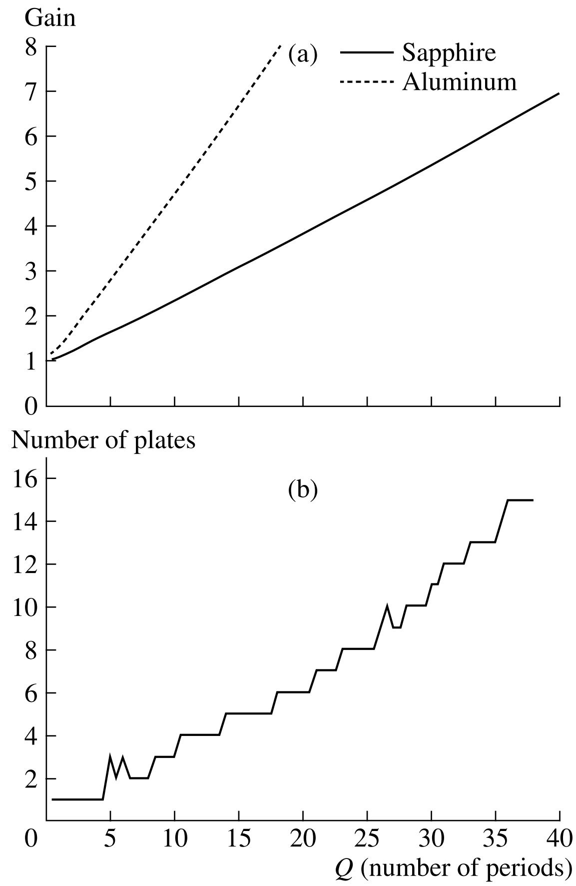

Fig. 5. (a) Maximum gain and (b) number of plates necessary to achieve this gain versus the duration (number of periods Q) of the pulse being amplified.

Figure 5b shows the theoretical number of plates required to achieve the gain shown in Fig. 5a. The plot is not smooth, because the number of plates can only be integer. The spikes near $Q = 5$ and 25 are caused by a certain ambiguity in the choice of the number of plates, the deviation being no greater than 1.

PULSE COMPRESSION USING A THREE-PLATE REFLECTOR

In the experiments with one plate, we managed to achieve an amplitude gain of 1.6. Theoretically, a higher gain can be obtained by increasing the number of plates (see Fig. 5). In particular, three plates can provide an amplitude gain of more than 2.

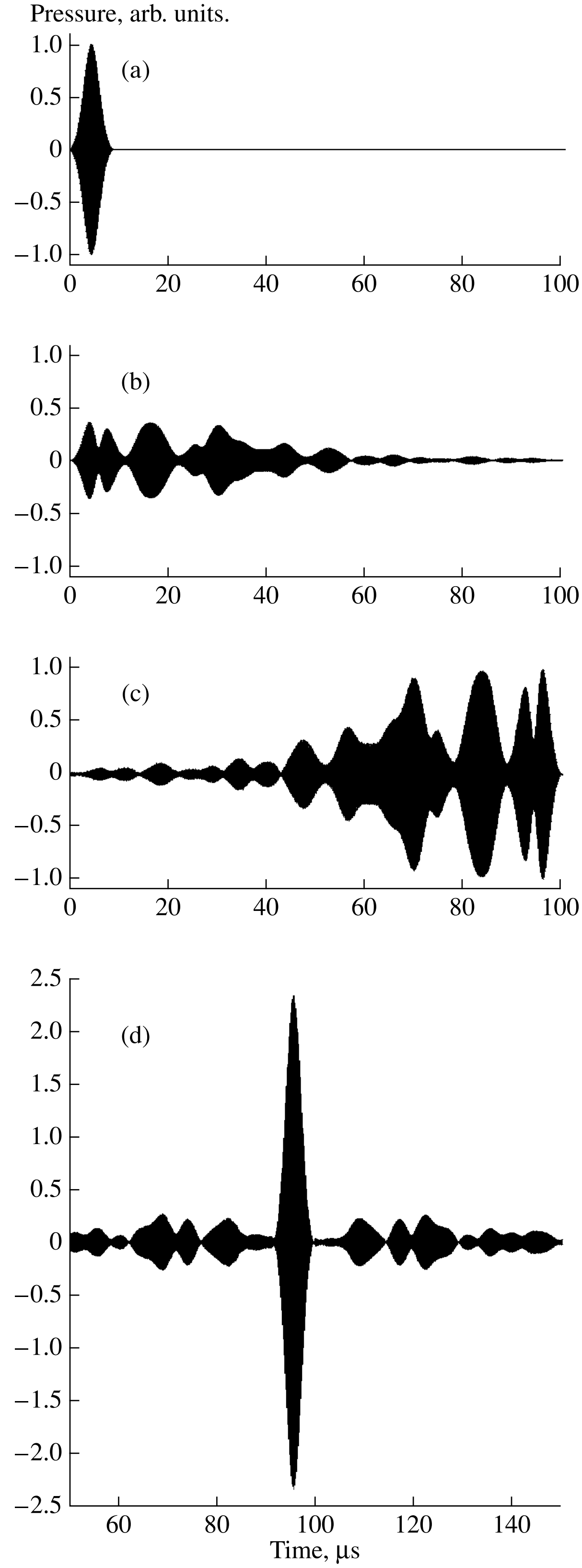

Fig. 6. Results of the experiment with the three-layer structure. The plots show the same types of signals as those in Fig. 2: (a) initial pulse, (b) reflected pulse, (c) time-reversed reflected pulse, and (d) compressed pulse.

The three-layer structure was composed of sapphire plates similar to those used in the experiment with one plate. The thicknesses of the plates were 0.98, 0.99, and 1.00 mm. The fact that the plates differed in their thickness allowed them to reflect different parts of the incident spectrum. The gaps between the plates were 6.8 and 7.5 mm; i.e., the total thickness of the multilayer reflecting structure was less than 2 cm. We have found theoretically that this structure can provide an amplitude gain of 2.7. The experimental results are shown in Fig. 6. It is seen that the short pulse, after being reflected from the three-layer structure, increased its duration (measured between the e^{-1} points) by a factor of 10. The amplitude decrease due to the reflection was slightly greater than the theoretical value, namely, by a factor of 2.8, which suggests that the system behaves not as perfect as in the experiment with one plate and that an energy loss occurs in this system. The signal obtained was time reversed and increased to the level of the original signal, and, subsequently, it was compressed and amplified. As a result, the signal was amplified by a factor of 2.3 and its waveform was adequately reconstructed. It is seen that, in addition to the main high-amplitude pulse, the compressed signal contains unwanted small-amplitude pulses. They appear because the loss in the system prevents us from exactly reconstructing the original signal's waveform. In our opinion, the loss appears mainly because the plates are not exactly parallel, which considerably distorts the amplitude of the recorded multiply reflected pulses. Additional errors are introduced by the diffraction effects, which were not taken into account in our model.

DISCUSSION

We compared the experimental results with those obtained by numerical simulation (Fig. 7). The comparison was performed for signals reflected from one-layer (Fig. 7a) and three-layer (Fig. 7b) structures. The comparison shows that the experimental results for the one-layer structure almost perfectly agree with those obtained with the help of the simplest model of a layered medium. For the three-layer structure, the agreement between experiment and theory is not as perfect: the amplitude of the measured signal is seen to be substantially lower than the theoretical value. This is particularly seen in the region beyond 30–40 μs. This region corresponds to multiply reflected pulses, which are most sensitive to inclination of the interfaces and are vulnerable to diffraction effects. At first sight, diffraction must not noticeably affect the results when the reflector is in the near-field zone of the transducer. However, the effective distance covered by the wave is in fact somewhat longer than twice the distance to the reflecting structure, because, in the process of multiple reflections inside the structure, the signal travels an additional path. As a result, an additional effect of diffraction divergence appears, which, along with the

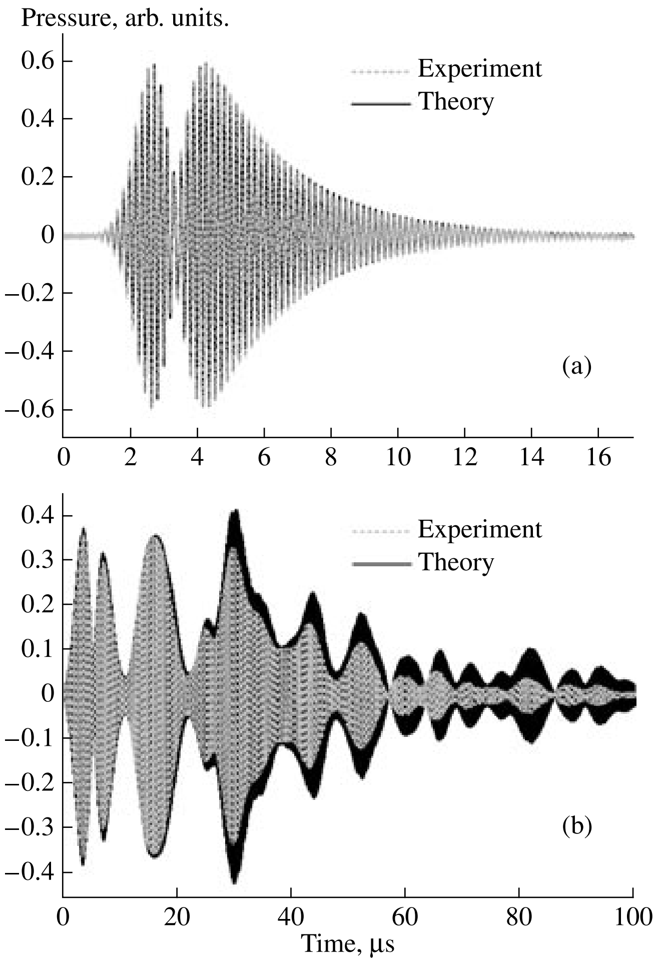

Fig. 7. Comparison of the experimental and theoretical reflected pulses for (a) one-layer and (b) three-layer structures.

effect of plate inclination, causes the experimental results for the three-layer structure to slightly differ from those predicted by the one-dimensional theoretical model.

CONCLUSIONS

Our study has shown that the simplest one-dimensional multilayer structures are capable of considerably increasing the amplitude of the wave due to the appropriate shortening of the signal. For the compression to be possible, the signal's waveform must be specially tailored to the particular multilayer structure. The experimental compressing structures consisting of one and three plates were found to be capable of increasing the signal amplitude by a factor of 1.6 and 2.3, respectively. An algorithm for designing efficient compression systems is proposed. A characteristic feature of these structures is that they not only contain inhomogeneous layers, but also that these layers have a specific spatial distribution of their reflection properties.

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REFERENCES

1. S. A. Akhmanov, V. A. Vysloukh, and A. S. Chirkin, *The Optics of Femtosecond Laser Pulses* (Nauka, Moscow, 1988) [in Russian].
2. M. Fink, Phys. Today **50**, 34 (1997).
3. V. A. Zverev, *Selected Works* (IPF RAN, Nizhni Novgorod, 2004) [in Russian].
4. V. A. Zverev, Akust. Zh. **50**, 792 (2004) [Acoust. Phys. **50**, 685 (2004)]].
5. A. P. Brysev, L. M. Krutyanskiĭ, and V. L. Preobrazhenskiĭ, Usp. Fiz. Nauk **168**, 877 (1998) [Phys. Usp. **41**, 793 (1998)].
6. A. P. Brysev, V. G. Mikhalevich, and V. N. Strel'tsov, Pis'ma Zh. Éksp. Teor. Fiz. **75**, 660 (2002) [JETP Lett. **75**, 543 (2002)].
7. B. Ya. Zel'dovich, N. F. Pilipetskiĭ, and V. V. Shkunov, *Phase Conjugation* (Nauka, Moscow, 1985) [in Russian].
8. V. V. Ragul'skiĭ, *Phase Conjugation at Stimulated Light Scattering* (Nauka, Moscow, 1990) [in Russian].
9. H. C. Song, W. A. Kuperman, and W. S. Hodgkiss, J. Acoust. Soc. Am. **103**, 3234 (1998).
10. D. R. Jackson and D. R. Dowling, J. Acoust. Soc. Am. **89**, 171 (1991).
11. N. Chakroun, M. Fink, and F. Wu, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **42**, 1087 (1995).
12. M. Pernot, J.-F. Aubry, M. Tanter, et al., Phys. Med. Biol. **48**, 2577 (2003).
13. G. T. Clement, J. Sun, T. Giesecke, and K. Hynynen, Phys. Med. Biol. **45**, 3707 (2000).
14. G. Montaldo, P. Roux, A. Derode, et al., J. Acoust. Soc. Am. **109**, 2481 (2001).
15. L. M. Brekhovskikh, *Waves in Layered Media* (Nauka, Moscow, 1973; Academic, New York, 1980).
16. G. S. Settles, *Schlieren and Shadowgraph Techniques* (Springer, Heidelberg, 2001).
17. L. Brillouin, *Wave Propagation in Periodic Structures* (Dover, New York, 1953; Inostrannaya Literatura, Moscow, 1959).

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