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Calculating an Acoustic Trap for an Elastic Spherical Scatterer of Large Wave Size

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Abstract—Modeling is performed for an acoustic trap created in a liquid using a multi-element ring transducer operating in a megahertz frequency band. A GUI service (service with a graphical interface) is created to calculate the acoustic radiation force acting on an elastic spherical scatterer of large wave size placed in the trap. The angular spectrum is used to calculate the radiation force by solving the complete scattering problem of an acoustic beam on a spherical scatterer. Results from a numerical experiment demonstrate the possibility of retaining solid particles and allow the accuracy of this technique to be determined.

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INTRODUCITON

Creating an acoustic trap [1, 2]—an area of space in which an acoustic beam can be used to retain a solid particle—is necessary in many applied problems. To ensure this configuration of an acoustic field is an acoustic trap for a certain particle, we must calculate the acoustic radiation force that acts on it. If the size of the particle considerably exceeds or is comparable to the wavelength, calculating the acoustic radiation force becomes a difficult computational problem that requires us to solve the problem of acoustic wave scattering because the existing simplifications for calculating the force (e.g., Gor'kov's approximation [3]) can only be used for scatterers of small wave sizes.

A way of calculating the radiation force by creating an arbitrary acoustic beam and acting on an elastic sphere in a liquid was proposed in [4]. The beam incident on the scatterer is described as the sum of plane waves of different directions (the angular spectrum). A classical solution to the problem of scattering on an elastic sphere is applied to each of these waves, and the resulting scattered field is presented as the superposition of scattered waves of all components of the angular spectrum of the incident beam. The sum of the incident and scattered waves allows us to calculate components of the radiation stress tensor on a closed surface surrounding the scatterer, and thus to calculate the acoustic radiation force.

Although the technique proposed in [4] allows us to calculate with good accuracy the radiation force for an arbitrary acoustic field and any material and size of a spherical scatterer (meaning it can be used in a wide range of practical problems), numerical implementation of this technique can be difficult, due to the complexity of the formulas and the need to consider a great many parameters simultaneously. A GUI service in the Python language that allowed us to calculate the acoustic radiation force by setting parameters of the acoustic field and scatterer through a user-friendly interface was therefore used to make the computational procedure and analysis of the results easier.

The above module was used to show that the pattern of the ultrasonic beam considered below was an acoustic trap for a scatterer of a large wave size.

ACOUSTIC RADIATION FORCE CALCULATION METHOD

To calculate the acoustic radiation force, we must know the acoustic field in which a scatterer is placed. Let us consider an acoustic source that emits a wave beam along axis z. It is assumed that the source emits harmonic waves. A hologram in the form of a two-dimensional pressure distribution over plane xy in the front of the source characterizes the acoustic field. This distribution can be calculated using a model of a vibrating surface of a source and the Rayleigh integral, or it can be measured experimentally. We fix the intersection of the hologram plane and axis z as the origin of the Cartesian coordinate system. The distribution of the complex pressure amplitude of the incident wave in plane xy when $z = 0$ is thus known: $p_i(x, y, z = 0)$. Spectrum $S(k_x, k_y)$ is then also known, where k_x, k_y are the components of wavevector $k = \{k_x, k_y, k_z = \sqrt{k^2 - k_x^2 - k_y^2}\}$. The angular spectrum and the incident field of pressure $p_i(x, y, 0)$ are related by a two-dimensional Fourier transform over coordinates (x, y). The complex amplitude of the acoustic pressure in an arbitrary spatial point is determined from the angular spectrum as [5, 6]

$$p_i(x, y, z) = \frac{1}{4\pi^2} \iint\limits_{k_x^2+k_y^2<k^2} dk_x dk_y S(k_x, k_y) e^{ik_x x + ik_y y + i\sqrt{k^2-k_x^2-k_y^2}z}. \quad (1)$$

It was shown in [2] that if an acoustic beam falls on an elastic spherical scatterer, the sum of the incident and scattered fields of the complex pressure amplitude for a scatterer located at point $\vec{r} = \{x, y, z\}$ is written as the expansion

$$p = \frac{1}{\pi}\sum_{n=0}^{\infty} i^n \left\{ j_n(kr) + c_n h_n^{(1)}(kr) \right\} \sum_{m=-n}^{n} H_{nm} Y_{nm}(\theta, \varphi), \quad (2)$$

where j_n, $h_n^{(1)}$ are the spherical Bessel and Hankel functions, Y_{nm} is the spherical function, c_n are coefficient determined by characteristics of the scatterer and surrounding medium, H_{nm} denotes matrix coefficients characterizing the incident field, and $r = |\vec{r}|$. Coefficients H_{nm} can be found using angular spectrum $S(k_x, k_y)$ of the incident wave and complex conjugate spherical harmonics Y_{nm}:

$$H_{nm} = \int\limits_{k_x^2+k_y^2\le k^2} dk_x dk_y S(k_x, k_y) Y_{nm}^*(\theta_k, \varphi_k). \quad (3)$$

Note that H_{nm} is calculated for the angular spectrum that corresponds to the location of the scatterer at point $\vec{r} = \{0, 0, 0\}$. If the scatterer is placed at another spatial point $\vec{r} = \{x, y, z\}$, the corresponding angular spectrum is determined by multiplying the initial angular spectrum by the respective propagator

$$\tilde{S}(k_x, k_y) = S(k_x, k_y) e^{ik_x x + ik_y y + i\sqrt{k^2-k_x^2-k_y^2}z}. \quad (4)$$

Obtained coefficients H_{nm} are used to calculate the Cartesian coordinates of the vector of the acoustic radiation force acting on the scatterer at this spatial point:

$$F_x = \frac{1}{8\pi^2\rho c^2 k^2} \mathrm{Re}\left\{ \sum_{n=0}^{\infty} \psi_n \sum_{m=-n}^{n} A_{nm} \left(H_{nm} H_{n+1,m+1}^* - H_{n,-m} H_{n+1,-m-1}^* \right) \right\}, \quad (5)$$

$$F_y = \frac{1}{8\pi^2\rho c^2 k^2} \mathrm{Im}\left\{ \sum_{n=0}^{\infty} \psi_n \sum_{m=-n}^{n} A_{nm} \left(H_{nm} H_{n+1,m+1}^* + H_{n,-m} H_{n+1,-m-1}^* \right) \right\}, \quad (6)$$

$$F_z = -\frac{1}{4\pi^2\rho c^2 k^2} \mathrm{Re}\left\{ \sum_{n=0}^{\infty} \psi_n \sum_{m=-n}^{n} B_{nm} H_{nm} H_{n+1,m}^* \right\}, \quad (7)$$

where A_{nm}, B_{nm}, and ψ_n are written as

$$\psi_n = (1 + 2c_n)\left(1 + 2c_{n+1}^*\right) - 1, \quad (8)$$

$$A_{nm} = \sqrt{\frac{(n+m+1)(n+m+2)}{(2n+1)(2n+3)}}, \quad (9)$$

$$B_{nm} = \sqrt{\frac{(n+m+1)(n-m+1)}{(2n+1)(2n+3)}}. \quad (10)$$

Relations (2, 4–7) show that summation over n from zero to infinity can be done everywhere. The magnitude of the x-components of the radiation force actually falls quite quickly, and the declining rate depends especially on multiplier ψ_n, which is a function of the wave size of the scatterer ka, where a is the scatterer's radius. The higher the value ka, the greater the number of summation terms must be considered for accurate calculations of the acoustic radiation force. When $ka < 1$, only two terms of the series are required, and the expressions are reduced to Gor'kov's approximation for scatterers of a small wave size [3].

ESTIMATING THE ACCURACY OF CALCULATIONS

The time needed for calculations depends quadratically on the number of iterative terms and linearly on the number of spatial points. The optimum running time of the algorithm therefore requires that we determine the minimum number of retained terms of the series that will yield acceptable accuracy of calculation. The number of terms of the series was limited in our algorithm by value $N_{\max} = ka + 5$, where k is the wavenumber in the medium and a is the radius of the scatterer. It was found that the numerical error of the calculations did not in this case exceed 10^{-5}.

We can derive F_z from expression (7) as a function of upper summation index N:

$$F_z(N) = \sum_{n=0}^{N} f_n, \quad (11)$$

where f_n denotes the n-th components from (7). We then consider number N of terms of the series as suffi-

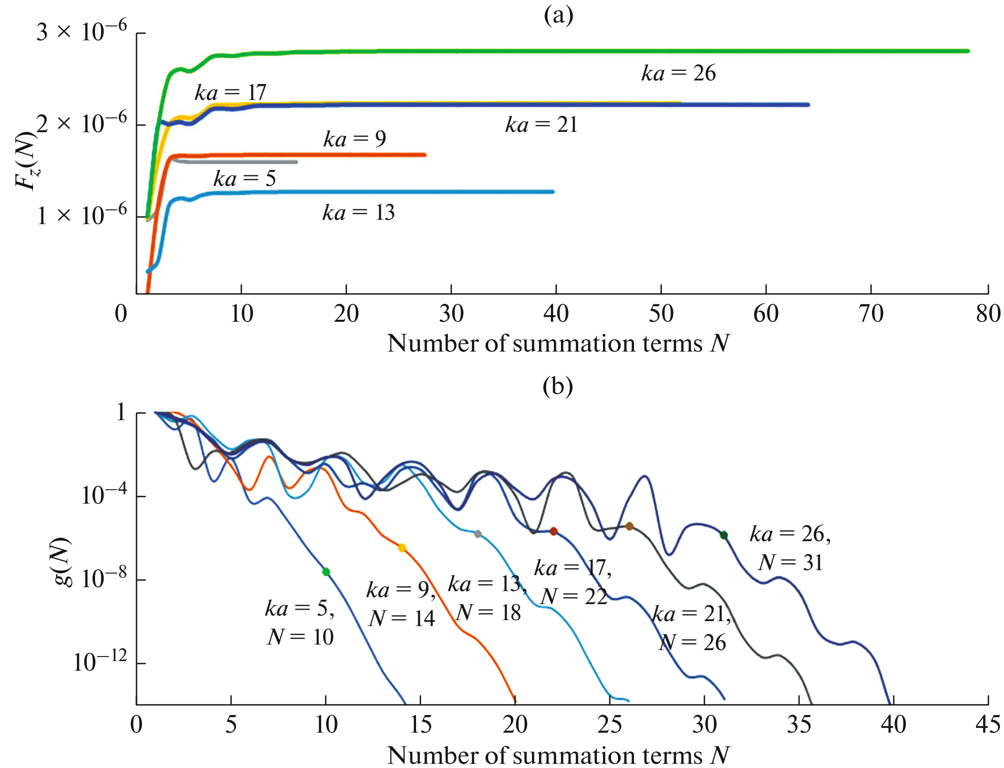

Fig. 1. Dependences (a) $F_z(N)$ and (b) $g(N)$ for each size of a scatterer. It is shown that number $N_{max} = ka + 5$ of terms of the series to be retained is sufficient for high accuracy of numerical calculations of the acoustic radiation force to be obtained for scatterers of different sizes.

cient if $F_z(N)$ approaches an asymptote, and the value of $|F_z(N-1) - F_z(N)|/F_z(N)$ does not exceed 10^{-5}.

As an example, we calculated the values of the z-component of the radiation force acting on a spherical scatterer of different wave sizes positioned at the focus. We estimated the accuracy by modeling the source of the field as a concave focused transducer in a shape of a spherical cup with diameter 10 cm, a radius of curvature of 7 cm, and an operating frequency of 1 MHz. The scatterer's characteristics were density $\rho_{sc} = 2040$ kg/m^3; velocities of longitudinal and shear waves $c_l = 4540$ and $c_t = 2130$ m/s, respectively; and a 1 to 6 mm radius of the scatterer, which corresponded to wave size ka of 5 to 26 (i.e., a large wave size scatterer). The scatterer characteristics corresponded to calcium oxalate monohydrate used in clinical trials for the ultrasonic expulsion of kidney stones.

Figure 1a shows dependence $F_z(N)$ for each size of the scatterer. We can see that the lower the wave size the faster F_z approaches the asymptote. We plotted dependences $g(N) = 1 - \left|\frac{F_z(N-1)}{F_z(N)}\right|$ (Fig. 1b) to estimate number N of terms of the series that must be retained for required accuracy of calculation g to be achieved. We can see that $N_{max} = ka + 5$ (indicated at each plot by points) resulted in a calculation error of 10^{-5} for all sizes of the scatterer. Similar calculations for the transverse components of the acoustic radiation force also showed that this criterion is sufficient for N_{max}.

CALCULATING AN ACOUSTIC TRAP FOR A LARGE WAVE SIZE SCATTERER

To keep a rather large scatterer in a certain spatial domain, we propose creating a pattern of the sound field in the form of a cup with the minimum amplitude of acoustic pressure in the center and the maximum at the edges. The characteristic frequency of the ultrasonic beam is assumed to be 2 MHz, while the characteristic sizes of the scatterer can be as high as several millimeters in radius. The zone of the acoustic trap must therefore be quite wide. In this formulation of the problem, we decided to form the walls of the acoustic trap cup with a focal curve by choosing an appropriate phase of the signal for each element of the transducer.

A hemispherical focal curve can be created by creating a radiating surface with a curvature correspond-

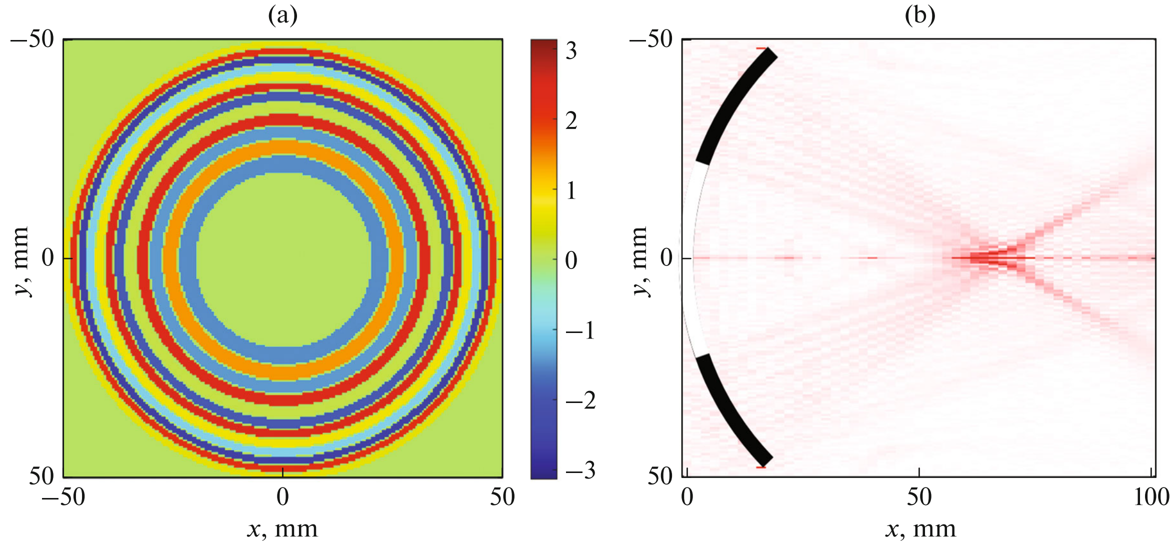

Fig. 2. (a) Phase distribution on the transducer elements and (b) corresponding field of the acoustic pressure forming a cup-shaped focal curve as the acoustic trap for a scatterer of a large wave size.

ing to the evolvent of the circle. Rays emitted perpendicular to the radiating surface form a hemispherical focal curve with radius *r*, and the parametric equation of the evolvent is determined by the equations

$$x = r(\cos\phi + \phi\sin\phi), \quad (12)$$

$$y = r(\sin\phi - \phi\cos\phi), \quad (13)$$

where *r* is the radius of the focal curve periphery and ϕ is the polar angle of the point of contact between the tangent line and the periphery.

To form a cup-shaped focal curve, we must therefore use either a transducer with a surface curvature determined by evolvent equations or a multi-element transducer and fix the corresponding wavefront by phasing the radiating elements. The latter is better when solving practical problems.

To form an acoustic trap 1 cm in width, we calculated the phase for elements of an existing ring multi-element transducer that contained 12 rings with radii of 2 to 5 cm and had a radius of curvature of 7 cm. Figure 2a shows the distribution of phases in the emitted signal at each element of the transducer. Figure 2b shows the pattern of the resulting field, calculated using the Rayleigh integral for radiation of an ultrasonic beam at a frequency of 2 MHz into water. This pattern shows that phasing the elements of the transducer created a focal curve in the form of a cup in which a scatterer of a fairly large wave size can be held.

To prove that the created field was an acoustic trap, we must calculate the acoustic radiation force acting on a scatterer placed in this trap. An original GUI service was used that allowed us to calculate the acoustic radiation force as described above using a hologram of the acoustic beam and the characteristics of the scatterer.

CALCULATING THE ACOUSTIC RADIATION FORCE

The above service was created in the Python 3 programming language, while the graphic interface was implemented using the Flask web development framework and is available in any browser. PostgreSQL is used as a database that stores the results from calculations and uploaded models of the acoustic beam. There is also a Docker image of this module that can be run on any operating system or cloud server.

The calculated theoretically using the Rayleigh integral hologram of the ultrasonic beam [5] was used as the source of the field and represented the distribution of the complex amplitude of the pressure in the plane normal to the direction of the beam propagation. It was in the form of a table containing $N \times N$ elements in the .mat format. The geometries of the transducer and hologram plane are shown in Fig. 3. The experimentally measured hologram of the same transducer can also be used in practical implementations as a source of information about the ultrasonic beam, allowing us to calculate the acoustic radiation force more accurately.

To demonstrate the efficiency of the theoretical model, a steel ball with a radius of 1 mm was considered as a scatterer. The velocity of longitudinal acoustic waves was 5740 m/s. That of shear waves was 3092 m/s, and the density was 7800 kg/m^3.

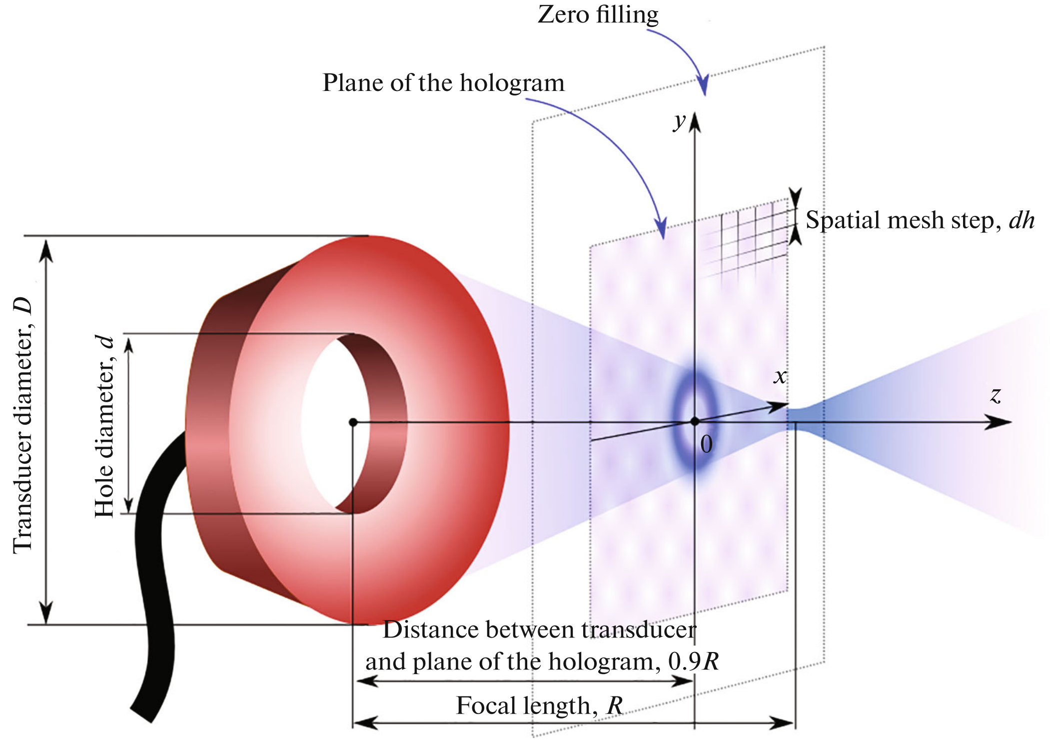

Fig. 3. Schematic image of the plane of calculating the hologram of a focusing transducer.

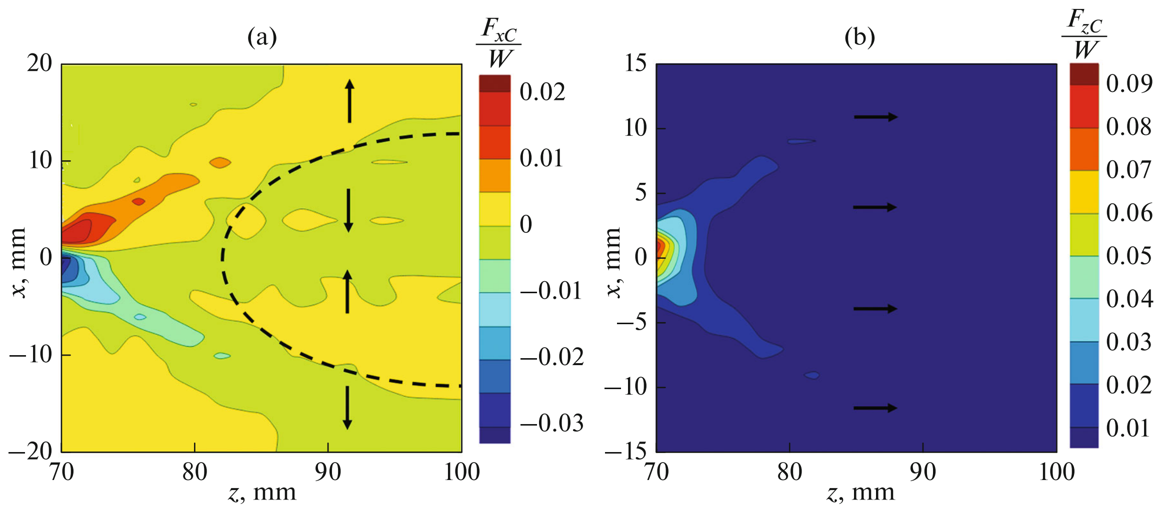

Fig. 4. Normalized (a) x-component and (b) z-component of the radiation force in plane xz acting on a steel spherical scatterer with a radius of 1 mm. The dotted curve indicates the zone of the acoustic trap; arrows show the directions of the x- and z-components of the acoustic radiation force.

RESULTS

The results obtained after calculating the acoustic radiation force with the original module are presented in the form of interactive plots in Fig. 4. Components of the radiation force were normalized by sound field power W and sound velocity in the medium c. Normalization of the radiation force is necessary when the amplitude of the pressure on the transducer surface is not known for certain, but the source power was measured via, e.g., acoustic weighing and its power is known. Normalization is performed using formula $F_{\text{norm}} = Fc/W$.

The distribution of the x-component of the acoustic radiation force in plane xz (Fig. 4) shows that the transverse component of the radiation force pulls the scatterer toward the beam axis in the zone of the acoustic trap. The longitudinal z-component is always positive, acts on the scatterer toward the direction of the beam propagation, and can be balanced by gravity acting on the scatterer. The proposed configuration of the field can therefore be an effective acoustic trap for scatterers of large wave size.

CONCLUSIONS

The configuration of a field was calculated that allowed us to create an acoustic trap for a scatterer of large wave size by compensating for the axial component of the force by gravity. This was demonstrated using an original GUI service and open code hosted on the GitHub platform [7]. The accuracy of our way of calculating the radiation force was determined.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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