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Nonlinear Acoustics**Oxford, UK
4-8 July 2022**Physical Acoustics: Session 4 - Radiation Force****Direct measurement of the radiation force of a
focused acoustic beam on a spherical particle in
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The purpose of this paper is to present a method for direct measurement of the acoustic radiation force exerted by an ultrasonic beam on a millimeter-sized spherical particle in a liquid and to compare the measurements with theoretical predictions. A focused ultrasonic beam was generated by a concave piezoceramic transducer with a center frequency of 1.072 MHz, a focal length of 70 mm, and an aperture of 100 mm. The transducer was placed face down in a water tank. A spherical bead was fixed under the source in the center of a plastic ring with the help of stretched fishing lines. The ring was attached to a rigid frame, which rested on an electronic balance without touching the tank. A thin membrane was placed above the bead to dampen the acoustic streaming. A positioning system was used to place the bead at different points on the transducer axis. The radiation force measurements were in good agreement with theoretical predictions. Some discrepancy occurred when the bead was close to the focal point, which was caused by the appearance of standing waves. Proper selection of the electrical impedance of the transducer helped reduce this artifact.

1. INTRODUCTION

The acoustic radiation force (ARF) enables to remotely manipulate small objects.^{1–3} Such an approach could be useful in various applications; e.g., in medicine, it is currently being investigated to expel small stones or residual fragments from the kidney.^{4,5} Accurate measurement of the radiation force acting on a millimeter-sized object is of considerable interest both for testing theoretical models and for elucidating the possibilities of using the radiation force effect in practice. Previously, several methods were proposed to measure the magnitude of the transverse and axial components of the ARF acting on a millimeter-sized sphere from an ultrasound beam.^{6–8} Despite the fact that these methods demonstrate a general agreement between theory and experiment, they are limited in accuracy (10–20%) and are quite laborious and time-consuming. In the current paper, a new method is proposed for measuring the acoustic radiation force exerted by an ultrasonic beam on an elastic sphere (a solid bead) in a liquid. The bead was suspended in the water with the help of thin fishing lines stretched inside a rigid frame, one part of which was in the water, and the other one, without touching the walls of the water tank, was placed on an electronic scale located under the tank. The value of the vertical component of the radiation force was determined by the results of weighing, similarly to the radiation force balance method of measuring the total power of an acoustic beam incident on an extended absorber.⁹ The measurements were compared with the theoretical value of the radiation force calculated using a previously developed approach based on the decomposition of the incident acoustic field into its angular spectrum.¹⁰ One of the key problems in previous force measurement methods was the strong influence of standing waves arising between the surfaces of the scatterer and the piezoelectric transducer.⁸ In this paper, to bypass this artifact, we use an approach based on the suppression of the reflection from the piezoelectric plate due to the proper change in the impedance of the electrical load of the transducer.¹¹

2. METHODS

A. ACOUSTIC RADIATION FORCE MEASUREMENTS

The experimental setup designed for measurements of vertical component of the ARF using the balance method is depicted in Fig. 1. A concave piezoceramic source having the shape of a spherical bowl with a radius of curvature (focal length) of 70 mm and a diameter of 100 mm, was placed in a tank filled with degassed water. The source was fixed in a positioning system, which made it possible to move it along three mutually perpendicular axes with a positioning accuracy of 2.5 μm . The axis of the radiated acoustic beam was directed vertically downward. A continuous harmonic signal at a frequency of 1.071 MHz was supplied from a generator (Agilent 33250A, Agilent Technologies, USA) through an amplifier (210L, Electronics & Innovation, USA). A spherical target (a bead) was positioned at some distance from the source on the acoustic beam axis, in the center of an auxiliary wide plastic ring. The bead was suspended using a system of stretched thin fishing lines with a diameter of 35.7 μm (NanoFil, Berkley, USA). The plastic ring was attached to a rigid frame, the shape of which made it possible to go around the water tank without touching its walls and transfer the force applied to the target directly to the surface of an electronic balance (VI-3mg, Acculab, USA). The precision of the balance and the scale division value were 4 mg and 1 mg, correspondingly. The balance readings during ARF measurements ranged from 10 to 650 mg. A thin polyethylene film with a thickness of less than 30 μm , used as a thin sound-transparent membrane, was stretched above the scatterer in order to suppress the influence of hydrodynamic flows initiated by the ultrasound beam. An acoustic absorber was located at the bottom of the tank to eliminate reflected waves. The signal supplied to the source was measured and controlled using an oscilloscope (TDS5034B, Tektronix, USA).

A stainless-steel sphere with a radius of $a = 3$ mm was used as a target. The target material parameters (the density ρ_* and speed of the longitudinal c_l and transverse c_t waves) were determined earlier from the scattering characteristics of an ultrasound beam and were as follows: $\rho_* = 7710$ kg/m³, $c_l = 5900$ m/s, and $c_t = 3245$ m/s.¹² To measure the radiation force at various points on the axis of the acoustic beam, the transducer was moved by the positioning system along the vertical z axis with a step of 0.1 or 0.25 mm.

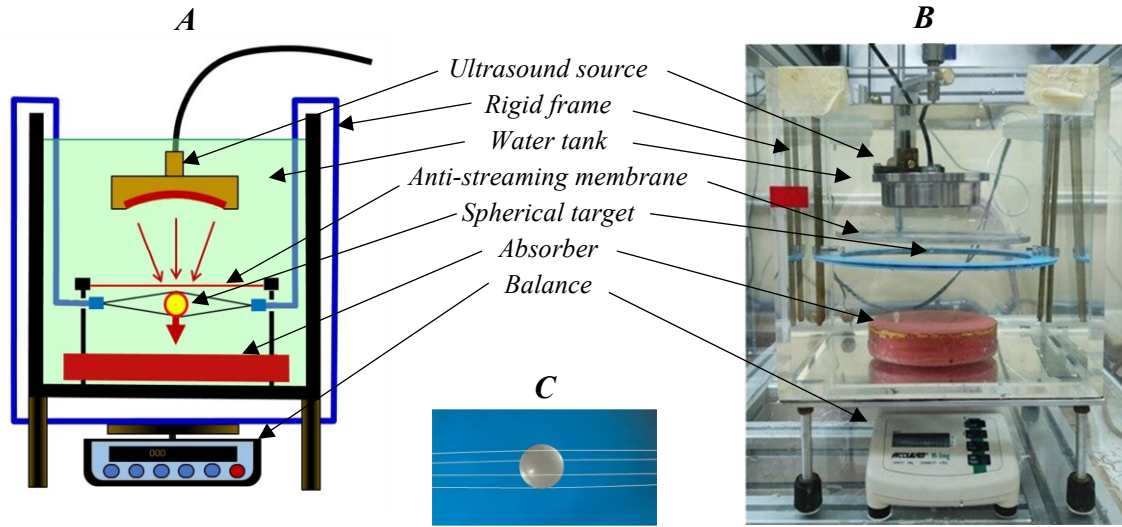


Figure 1. A) Experimental setup. B) Photo of the experimental arrangement. C) Fixing the scatterer with a system of stretched lines.

The magnitude of the radiation force was determined by the difference (m_{rad}) in the balance readings after and before the electrical signal was applied to the transducer from the generator. After the signal was turned on, the readings of the balance reached a constant value after 2 – 5 seconds on average. During this time, the oscillations of the bead, which arose after the impact due to the launch of ultrasound, faded, as evidenced by the establishment of a stable voltage signal on the transducer, observed on the oscilloscope. After turning off the voltage of the transducer, the balance readings were compared with those before the ultrasound was turned on. The discrepancy between the readings of the scales before and after turning on the ultrasound indicated the influence of artifacts (for example, sticking of bubbles to structural elements immersed in water). In this case, measurements at this point were repeated. To verify the reliability of the method, repeated measurements were carried out at some points both inside and outside the focal region. Several ARF measurements were taken at different times: repeated measurements gave the same result with an accuracy of 1 mg, i.e., reproducibility of measurements under constant conditions has been achieved.

In addition to the radiation force measurements, the structure of the ultrasound field was found using the acoustic holography method.¹³ The acoustic beam hologram was recorded with a calibrated needle-type hydrophone (HNA-0400, Onda, Sunnyvale, CA, USA). The hologram was then used to determine the angular spectrum and corresponding total acoustic power,¹⁴ which are necessary for calculating the radiation force.

Using the formula

$$Y_z^{exp} = \frac{m_{rad} g c}{W} \quad (1)$$

the weighing result, m_{rad} , was transformed into a dimensionless radiation force. Here $g = 9.81 \text{ m/s}^2$ is the acceleration of gravity, $c = 1490 \text{ m/s}$ is the speed of sound in water, W is the total acoustic power of the incident ultrasound beam.

B. THEORETICAL MODEL

The numerical calculation of the ARF was carried out based on the theoretical approach described in Ref. 10. The main expressions used in the calculations are briefly presented below. Consider an ultrasound source emitting a monochromatic acoustic beam directed along the z axis. An elastic spherical scatterer of radius a is immersed in an ideal fluid and placed in front of the ultrasound source. The radiation force acting on the scatterer has three Cartesian components: $\mathbf{F}_{rad} = (F_x, F_y, F_z)$. The vertical component under consideration is determined by the following expression:

$$F_z = -\frac{1}{4\pi^2 \rho c^2 k^2} \operatorname{Re} \left[\sum_{n=0}^{\infty} \Psi_n \sum_{m=-n}^n B_{nm} H_{nm} H_{n+1,m}^* \right]. \quad (2)$$

Here c is the speed of sound in the liquid (water), ρ is the liquid density, $k = \omega/c = 2\pi f/c$ is the wave number, f is the frequency, $\operatorname{Re}[\cdot]$ indicates the real part of the term in brackets, $B_{nm} = \sqrt{(n+m+1)(n-m+1)/(2n+1)(2n+3)}$ are the known numerical coefficients, and

$$H_{nm} = \iint_{k_x^2 + k_y^2 \leq k^2} dk_x dk_y S(k_x, k_y) Y_{nm}^*(\theta_k, \varphi_k) \quad (3)$$

are the coefficients that fully describe the incident field, which has an angular spectrum $S(k_x, k_y)$. Also $Y_{nm}(\theta, \varphi)$ are spherical harmonics; $\Psi_n = (1 + 2c_n)(1 + 2c_{n+1}^*) - 1$ are the coefficients that characterize scattering and depend on the known properties of the liquid and the bead material, namely, on the speed of sound and density of the liquid, the density of the sphere material, and the velocities of longitudinal and transverse waves in the sphere. The full forms of the coefficients are given in Ref. 10.

It is convenient to introduce the dimensionless expression for the radiation force normalized to the total acoustic beam power W (see also Eq.(1)):

$$Y_z = \frac{F_z c}{W}. \quad (4)$$

The total acoustic beam power W can be calculated from the known angular spectrum:^{10, 15}

$$W = \frac{1}{8\pi^2 \rho c} \iint_{k_x^2 + k_y^2 \leq k^2} dk_x dk_y |S(k_x, k_y)|^2 \sqrt{1 - \frac{k_x^2 + k_y^2}{k^2}} \quad (5)$$

C. COMPENSATION FOR SOUND REFLECTION FROM A PIEZOELECTRIC TRANSDUCER

As mentioned earlier, when measuring the radiation force acting on a spherical scatterer from a focusing piezoelectric transducer, the standing waves, which are generated between the surfaces of the transducer and the sphere, become significant.⁸ In this paper, a new approach is used to reduce the influence of emerging standing waves. It is based on minimizing the reflection coefficient from the surface of the piezoelectric transducer by selecting the electrical impedance of its load. This results in a decrease in the efficiency of standing waves generation, which in turn reduces their influence on the results of the ARF measurements.

Previously, the similar method was presented in relation to a flat transducer with an air backing and a flat reflector.¹¹ It was shown that in the vicinity of frequencies at which the imaginary part of the piezoelectric impedance vanishes (e.g., near the frequency of antiresonance) it is possible to suppress the sound reflection from a piezoelectric transducer with an air backing using a selected purely active resistance. In the case of an incident acoustic wave of an arbitrary frequency, at which the imaginary part of the transducer impedance does not fall to zero, it is also necessary to use reactance to minimize the reflection coefficient.

In the current work the abovementioned method was applied to the problem of minimizing the reflection coefficient from a piezoelectric transducer of large wave dimension in the form of a spherical bowl with an air backing, which is used in experimental measurements of the radiation force. An active impedance (a resistance) was considered as an electrical load, which allows to reduce the reflection

coefficient from the piezoelectric transducer near the frequency of antiresonance. A study was conducted to analyze the value of the coefficient of reflection from a concave piezoelectric transducer of interest depending on the frequency and the value of the active impedance of the transducer electrical load connected to the piezoceramic bowl, while a stainless-steel bead with a radius of 3 mm was used as a scatterer (reflector). This study was performed similarly to the experimental technique described in Ref. 11. It was found that the minimum of the reflection coefficient equaled to approximately 2% was achieved at a frequency of 1.071 MHz for the electrical load of 20 Ohm connected to the transducer, which corresponded to the connection of a 33-Ohm resistor in parallel to the generator. Consequently, by connecting an additional resistor of a selected value in parallel to the generator, it was possible to decrease the coefficient of reflection of waves scattered by the bead, and to reduce the influence of standing waves on the results of measurements of the radiation force. Therefore, to improve the accuracy of measuring the radiation force, the electrical part of the experimental setup proposed above was modified: a resistor with a selected active resistance (metal oxide film 33 Ohm resistor, MOX Jantzen metallized, 5 W, 5%, Denmark) was connected in parallel to the transducer in order to decrease the reflection of acoustic waves from the surface of the piezoceramic plate.

3. RESULTS

An experiment was carried out for the stainless-steel sphere at the operating frequency of 1.071 MHz. Figure 2 shows the results of measurements of the vertical component of the ARF (yellow curve) as a function of the distance from the source on the acoustic axis z and their comparison with the numerical calculations (blue curve). Significant discrepancies between experimental and theoretically calculated force values become visible in the focal area. Due to the generation of standing waves in the region between the surface of the source and the scatterer, oscillations appear, the amplitude of which near the focus reaches 30 – 50% of the magnitude of the ARF. To reduce the amplitude of these oscillations, an additional resistance of 33 Ohm was connected in parallel to the generator, and similar measurements of the ARF were carried out (red curve). It can be seen that using of an active resistance of 33 Ohm noticeably reduced the oscillations: by a factor of 3.7 in the focal area and by a factor of 2 at some distance from the focus. Despite the absence of complete suppression of standing waves, a considerable decrease in the amplitude of oscillations was achieved in the focus region, where standing waves generate most efficiently. Therefore, the proposed method, involving the selection of the electric load of the piezoelectric transducer to reduce the influence of emerging standing waves, can also be used for a spherical source.

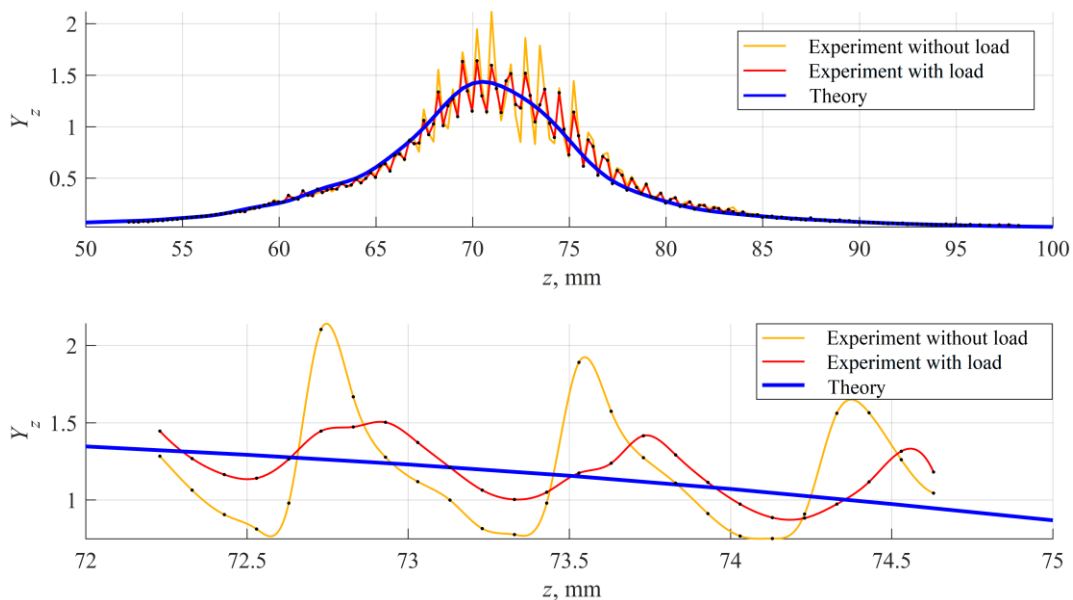


Figure 2. Axial distribution of the normalized radiation force Y_z acting on a stainless-steel sphere of radius $a = 3$ mm. The displacement step of the transducer was 0.25 mm (top) and 0.1 mm (bottom).

4. CONCLUSIONS

A balance method for measuring the vertical component of the ARF acting on a mm-sized solid particle was proposed. An accuracy of 30 – 40 μN was achieved. The ARF measurements were found to be in a good agreement with the theoretical predictions. Some discrepancy in the form of distance-dependent oscillations occurred when the bead was close to the focal point, which was caused by the appearance of standing waves between the surfaces of the scatterer and the transducer. Proper selection of the electrical load of the transducer helps to reduce this artifact and thus measure ARF with higher accuracy.

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REFERENCES

- ¹ L. V. King, “On the acoustic radiation pressure on spheres,” *Proc. R. Soc. London, Ser. A* **147**, 212–240 (1933).
- ² W. L. Nyborg, “Radiation pressure on a small rigid sphere,” *J. Acoust. Soc. Am.* **42**, 947–952 (1967).
- ³ L. P. Gor’kov, “On the forces acting on a small particle in an acoustic field in an ideal fluid,” *Sov. Phys. Dokl.* **6**, 773–775 (1962).
- ⁴ M. R. Bailey, Y.-N. Wang, W. Kreider, J. C. Dai, B. W. Cunitz, J. D. Harper, H. Chang, M. D. Sorensen, Z. Liu, O. Levy, and B. Dunmire, “Update on clinical trials of kidney stone repositioning and preclinical results of stone breaking with one system,” *Proc. Mtgs. Acoust.* **35**(1), 020004-1–020004-6 (2018).
- ⁵ M. A. Ghanem, A. D. Maxwell, Y.-N. Wang, B. W. Cunitz, V. A. Khokhlova, O. A. Sapozhnikov, M. R. Bailey, “Noninvasive acoustic manipulation of objects in a living body,” *Proc. Natl. Acad. Sci. U.S.A.* **117**(29), 16848–16855 (2020).
- ⁶ X. Chen and R. E. Apfel, “Radiation force on a spherical object in an axisymmetric wave field and its application to the calibration of high-frequency transducers,” *J. Acoust. Soc. Am.* **99**, 713–724 (1996).
- ⁷ M. A. Ghanem, A. D. Maxwell, O. A. Sapozhnikov, V. A. Khokhlova, M. R. Bailey, “Quantification of acoustic radiation forces on solid objects in fluid,” *Phys. Rev. Applied.* **12**, 044076-1–044076-13 (2019).
- ⁸ A. Nikolaeva, M. Karzova, S. Tsysar, V. Khokhlova, O. Sapozhnikov, “Experimental study of radiation force by a focused ultrasound beam on an elastic scatterer in a fluid,” *Proc. Mtgs. Acoust.* **38**(1), 045009-1–045009-5 (2019).
- ⁹ S. Maruvada, G. R. Harris, B. A. Herman, R. L. King, “Acoustic power calibration of high-intensity focused ultrasound transducers using a radiation force technique,” *J. Acoust. Soc. Am.* **121**(3), 1434–1439 (2007).
- ¹⁰ O. A. Sapozhnikov, M. R. Bailey, “Radiation force of an arbitrary acoustic beam on an elastic sphere in a fluid,” *J. Acoust. Soc. Am.* **133**(2), 661–676 (2013).
- ¹¹ L. M. Kotelnikova, A. A. Krokhmal, D. A. Nikolaev, S. A. Tsysar, O. A. Sapozhnikov, “Controlling the coefficient of reflection of sound from a plane piezoelectric plate by selecting its electrical load,” *Bull. Russ. Acad. Sci. Phys.* **85**(12), 1501–1506 (2021).
- ¹² L. M. Kotelnikova, D. A. Nikolaev, S. A. Tsysar, O. A. Sapozhnikov, “Determination of the elastic properties of a solid sphere based on the results of acoustic beam scattering,” *Acoust. Phys.* **67**(4), 360–374 (2021).
- ¹³ O. A. Sapozhnikov, S. A. Tsysar, V. A. Khokhlova, and W. Kreider, “Acoustic holography as a metrological tool for characterizing medical ultrasound sources and fields,” *J. Acoust. Soc. Am.* **138**(3), 1515–1532 (2015).
- ¹⁴ D. A. Nikolaev, S. A. Tsysar, V. A. Khokhlova, W. Kreider, and O. A. Sapozhnikov, “Holographic extraction of plane waves from an ultrasound beam for acoustic characterization of an absorbing layer of finite dimensions,” *J. Acoust. Soc. Am.* **149**(1), 386–404 (2021).
- ¹⁵ S. Tsysar, W. Kreider, O. Sapozhnikov, “Improved hydrophone calibration by combining acoustic holography with the radiation force balance measurements,” *Proc. Mtgs. Acoust.* **19**(1), 055015-1–055015-6 (2013).