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Application of a Mach–Zehnder Interferometer to the Observation of Mach Stem Formation When a Shock Wave is Reflected from a Rigid Surface

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Abstract—The reflection of a nonlinear spark-generated N-wave in air from a flat rigid surface is investigated experimentally. The pressure profile of the N-wave is reconstructed from optical measurements performed using a Mach–Zehnder interferometer. An irregular reflection is observed in the experiment; the evolution of a Mach stem is investigated, and the trajectory of a triple point during the N-wave propagation along the surface is investigated.

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INTRODUCTION

Reflection of shock waves from surfaces was first studied experimentally by Ernst Mach in 1878 [1]. Mach's experiments indicated that the law of mirror reflection where the incident and reflected angles are equal, is violated in the case of strong shocks. Moreover, a three shock structure, in which the incident and reflected shocks intersected above the surface and merged into one shock on the surface, was observed experimentally. Such interaction of shock waves was termed as irregular reflection, while the shock that forms near the surface was called the Mach stem in honor of Ernst Mach. The point where three shock fronts intersect was termed the triple point.

Theoretical investigations of irregular reflection were first carried out by von Neumann in 1940 [2]. The three-shock theory developed by von Neumann describes the irregular reflection only for strong shock waves with acoustic Mach numbers $M_a > 0.47$ [3]. For weak shocks ($M_a < 0.035$) the three-shock theory conflicts with experimental results and predicts that irregular reflection becomes physically unrealistic. At the same time, experimental observations confirm the opposite [4]. The conflict between von Neumann's theory and the experimental data is known as the von Neumann paradox, which was first formulated by Birkhoff in 1950 [5]. Numerous studies have been undertaken to resolve von Neumann paradox, mainly in the framework of aerodynamics covering step shock waveforms for acoustic Mach number greater than 0.03 [3, 4, 6–8]. Such waves are typical for aerodynamics, but not realistic for acoustics. In nonlinear acoustics, shock waves have more complicated waveforms than step shocks (e.g. N-waves and sawtooth waves) and the values of acoustic Mach numbers are at least one order smaller than in aerodynamics. The reflection of such very weak nonlinear acoustic waves has not been studied in detail. To our knowledge, the only experiment confirming the formation of a Mach stem for nonlinear acoustic waves was performed in [9] for the interaction of two sawtooth waves propagating in water. The authors of [10] investigated the reflection of acoustic shock waves from a rigid surface in numerical experiments for plane N-waves and sawtooth waves. The aim of this work was an experimental investigation of the reflection of a spherically divergent N-wave, generated by a spark source in air, from a flat rigid surface.

EXPERIMENTAL

A reflection pattern of an N-wave was measured in air using a Mach–Zehnder interferometer. The sketch of the experimental setup is shown in Fig. 1.

A spherically divergent N-wave was generated by a spark source with an applied voltage of 15 kV. The gap between the electrodes was 2 cm. A rigid surface was positioned under the spark source at a distance of h_{sp} = 21 mm. The probing laser beam of the interferometer, in which the phase shift φ was introduced by variations of the optical refraction index, was located at distance l from the spark source . The interferometer consisted of a He–Ne laser (λ = 632 nm, 10 mW), two beam splitters, and two mirrors arranged at a 45° angle to the beam; the signal was registered by a photodiode.

Measurements were performed as follows: when there was no acoustic wave, the interferometer was stabilized so that the output signal was equal to the sum of intensities of the reference and the probing beam: $I = I_1 + I_2$. When an acoustic wave passed through the

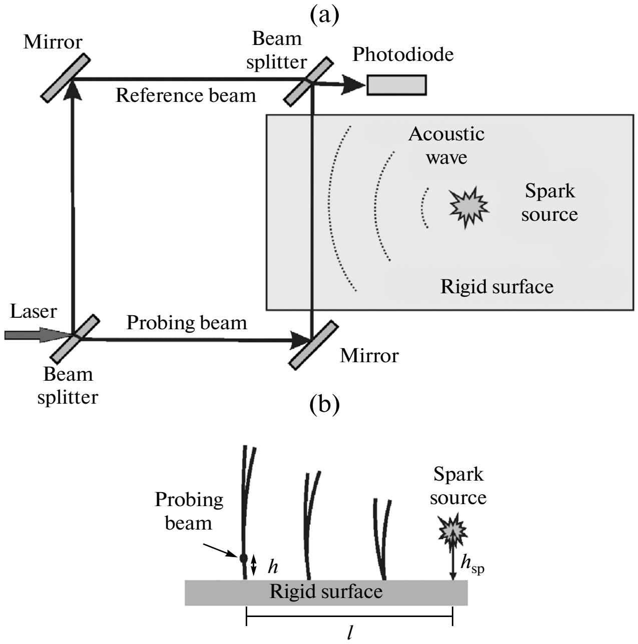

Fig. 1. Schematic of the experimental setup with a Mach–Zehnder interferometer: (a) top view and (b) side view onto the reflecting surface.

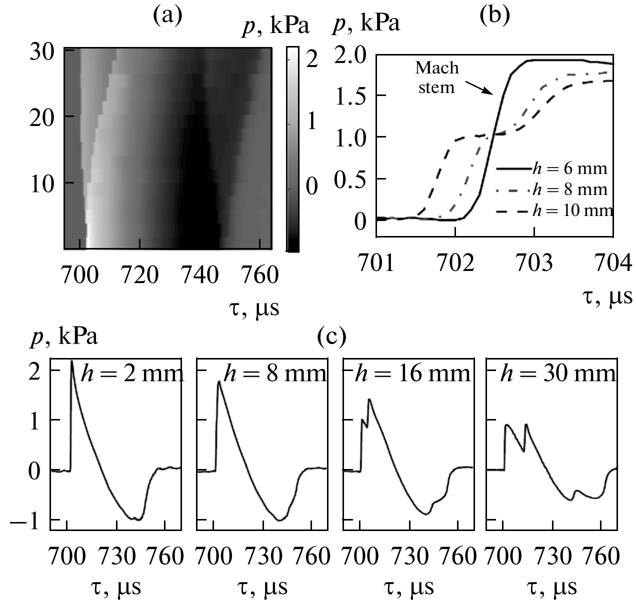

Fig. 2. Spatiotemporal characteristics of the acoustic field generated when an *N*-wave reflects from the surface: (a) reflection pattern, (b) structure of the leading front, and (c) reconstructed profiles of the *N*-wave at different heights *h* above the surface.

probing beam, light intensity formed by the interference of two beams at the photodiode surface was described by the following equation:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi. \quad (1)$$

Neglecting light refraction of the probing light beam and taking into account radial symmetry of the wavefront, the optical phase difference φ can be written as the direct Abel transform of the refractive index n:

$$\varphi(R) = \frac{4\pi}{\lambda}\int_r^{\infty}\frac{n(r)rdr}{\sqrt{r^2 - R^2}}, \quad (2)$$

where R is the distance from the spark source. Reducing the phase from optical signal (1) and applying the inverse Abel transform to expression (2), the refractive index n was reconstructed. The acoustic pressure in the wave was calculated with Gladstone relationship $p = nc_0^2/G$ [11], where c_0 is the speed of light in air and $G = 0.000226$ m^3/kg is the Gladstone constant when $\lambda = 632$ nm. As it was shown earlier in [12], the temporal resolution of the interferometer was 0.4 μs.

The reflection pattern reconstructed using the Mach–Zehnder interferometer is presented in Fig. 2a for distance $l = 25$ cm. This pattern represents the results from measuring wave profiles at distances h above the rigid surface in the range of 2 to 30 mm with steps of 2 mm. Profiles of 150 charges of the spark source, from which one average profile was later selected (i.e., the profile in which the values of the peak positive and negative pressures, and the time of wave arrival, were closest to the average values calculated for all of the spark charges). The pressure levels of the acoustic wave are indicated by different colors.

It is clear from Fig. 2a that the leading front of the *N*-wave is reflected from the surface in an irregular way. Only one front (the Mach stem) forms at distances $h \leq 6$ mm; starting at $h = 8$ mm, it splits into two fronts (of incident and reflected waves). The structure of the leading front in the vicinity of the triple point is shown in Fig. 2b, where the Mach stem separation into two fronts is observed. The irregular reflection was observed only for the front shock of the *N*-wave; the rear shock reflected regularly, since it was smother and had greater shock rise time. In the wave profiles shown in Fig. 2c for different heights h above the surface, the fronts of the incident and reflected waves (the profiles at $h = 2$ mm) were first observed in the structure of the front shock and subsequently diverged from one another.

We also measured the trajectory of a triple point during the Mach stem propagation along the surface (Fig. 3). The experimental data are well approximated by the linear dependence within the limits of distances l studied in the experiment. The distance h minimally possible for measurement was 2 mm; however, linear interpolation of this dependence in the direction of reducing the Mach stem predicts that the regular reflection becomes irregular at $l = 8$ cm. It should be noted that the results of numerical modeling presented

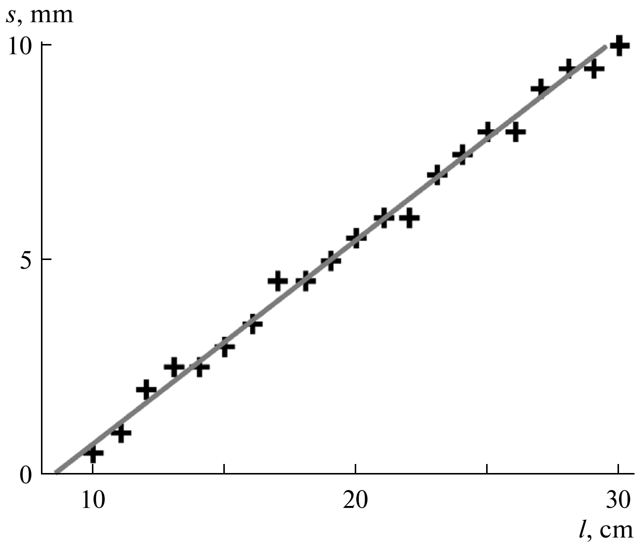

Fig. 3. Trajectory of a triple point: s is the height of the Mach stem and l is the distance along the surface from the spark source to the measuring point.

in [10] predict a nonlinear character (initially nonlinear growth and then decay) of variation in the length of the Mach stem during its propagation along a plane surface for an ideal plane N-wave. Whether our linear dependence is a consequence of the sphericity of wave propagation and its more rapid attenuation (compared to the plane wave studied in [10]), or our trajectory of the triple point is the initial segment of a more complex nonlinear dependence, are questions for further research.

CONCLUSIONS

The Mach stem formation in the process of spherically divergent N-wave reflection from a rigid surface was confirmed experimentally in air for acoustic Mach numbers in the range of $M_a \sim 10^{-2}$ to 10^{-1}.

(i) The use of a Mach–Zehnder interferometer provided pressure waveform reconstruction for spherically symmetric waves including the reflection from rigid surfaces. The temporal resolution of the proposed optical method is 0.4 μs, which is six times better than that of condenser microphones.

(ii) The height of the Mach stem increases linearly when a spherically divergent N-wave is reflected from a rigid surface. This effect was observed within the limits of the distances considered in our experiment (about fifteen lengths of the N-wave).

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